Numerical analysis of a spectral problem with high order boundary conditions on curved meshes

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Summary :
In this talk, we consider a spectral problem, referred to as the Ventcel eigenvalue problem, involving a second order term on the domain boundary (the Laplace-Beltrami operator, denoted Δ_Γ). Let Ω be a domain of $\mathbb{R}^d$ ($d = 2, 3$) with a smooth (at least $\mathcal{C}^2$) boundary $\Gamma := \partial\Omega$,

$$\begin{cases} -\Delta u = \lambda u & \text{in } \Omega, \\ -\Delta_\Gamma u + \partial_{\mathrm{n}} u + u = 0 & \text{on } \Gamma. \end{cases}$$

where $\partial_{\mathrm{n}} u$ is the normal derivative of u. The main objective of this work is to do an error analysis of the Ventcel eigenvalue Problem, estimating the error for the eigenvalues and the eigenfunctions. A crucial point concerns the construction of high order curved meshes for the discretization of the physical domain (see Figure 1) and on the definition of the lift operator defined in [1], which is aimed to transform a function defined on the mesh domain into a function defined on the physical one. This lift is defined in a way as to satisfy adapted properties on the boundary, relatively to the trace operator, which were not satisfied in [3] for a different lift definition. A bootstrap method is used to prove the error estimates, which are expressed both in terms of finite element approximation error and of geometrical error, respectively associated to the finite element degree $k \geq 1$ and to the mesh order $r \geq 1$ as detailed in [2]. The numerical experiments are led on various smooth domains in 2D and 3D in order to illustrate these results.

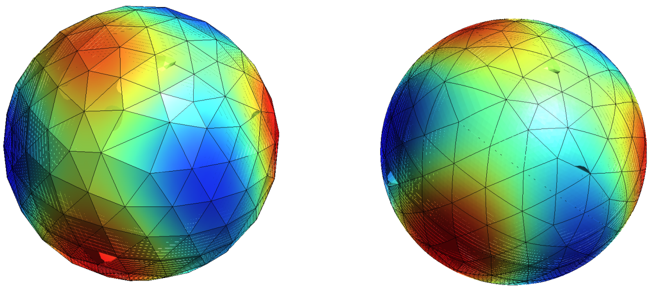

FIGURE 1 – Display of the eigenfunction associated with the eigenvalue Λ_{10} using $\mathbb{P}^2$ finite element method on an affine mesh (left) and a quadratic mash (right).

[1] F. Caubet, J. Ghantous, C. Pierre. *Numerical analysis of a diffusion equation with ventcel boundary condition using curved meshes.* (submitted), 2023.

[2] F. Caubet, J. Ghantous, C. Pierre. *Finite element analysis of a spectral problem on curved meshes occurring in diffusion with high order boundary conditions.* (submitted), 2024.

[3] C. M. Elliott, T. Ranner. *Finite element analysis for a coupled bulk-surface partial differential equation.* IMA J. Numer. Anal., **33(2)**, 377–402, 2013.

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