An incursion into volcanology

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Axe complexflows (PEPR MathsViVEs – France 2030).

Collaboration with

A. Burgisser and M. Collombet (ISTERRE - Le Bourget du Lac - France)
and
G. Narbona-Reina (Dpto Matematica Applicada - Sevilla - Spain)

Thanks to Catherine and Michel for the invitation

Thanks to the organizers

Île de ré, May 2024

Des outils pour les mathématiques en direction de la planète ?

Un peu d'histoire:
Document 2013 par D. Bresch et E. Neveu
Voir http://mathsmonde.math.cnrs.fr/

Par la suite avec Arnaud Guillin
(Directeur Fération maths Rhône-Alpes-Auvergne à l'époque),
Montage dossier pour CS INSMI pour création institut sans mur
$\Longrightarrow$ Recommandation du CS au CNRS.

Création (Mars 2021) de l'Institut Mathématiques de la planète terre : IMPT

GIS : Groupement d'Intérêt Scientifique (https://impt.math.cnrs.fr/)

Soutenu par : INSMI, INSU, ENS Lyon, Univ. Clermont Auvergne, Univ. Claude Bernard, Univ. Jean Monnet, Univ. Savoie Mont Blanc, Rejoint par Univ. Dauphine, Univ. Montpellier.

Direction: Laure Saint-Raymond
Direction adjoint: Arnaud Guillin

Bureau: Didier Bresch, Freddy Bouchet, Sabrina Speich,
François Munoz, Aude Pommeret, Amandine Veber

Comité scientifique: Une vingtaine de scientifiques internationalement reconnus toutes disciplines confondues réparti.e.s sur:

Terre humaine, Terre fluide, Terre vivante

- N'hésitez pas à faire vivre l'IMPT
  Partage d'informations, supports de cours etc....
- N'hésitez pas à pousser vos universités à soutenir l'IMPT.
- Appel à candidature par IMPT une fois par an pour demande subvention.
  Bourses thèses, post-doctorats, ...

PEPR Maths ViVEs (France 2030) dirigé par A. Guillin et V. Calvez

- 10 axes scientifiques sont subventionnés: Début rentrée 2024 sur 5 ans.
- Un appel à projet va étre ouvert vers Octobre 2024, via l'ANR
  et sur les thématiques générales du PEPR:
  Axe environnement, Axe Société, Axe Vivant.
- 10 autres axes seront choisis dans 3 ans.
- Un autre appel à projet dans 2 ans.

A long joint work with A. Burgisser, M. Collombet and G. Narbona-Reina with 3 papers almost accepted (revision form: almost 230 pages) to *Studies in Applied Maths* (MIT journal).

- Mathematical topics in compressible flows
  from single-phase systems to two-phase averaged systems (101 pages)
  –> State of the art on modeling of incompressible to compressible viscous flows ranging from single-phase to two-phase systems.
  –> Weak stability (important for numerical resolution) and homogenization process (important for averaged systems).
- Two-phase magma flow with phase exchange
  – Part I – Physical modeling of a volcanic conduit (78 pages)
  –> Steps needed at both microscopic and macroscopic scales to obtain a two-phase transient conduit flow model ensuring:
  - 1) Mass and volatile species conservation,
  - 2) Disequilibrium degassing
    considering both viscous relaxation and volatile diffusion,
  - 3) Dissipation of total energy.
- Two-phase magma flow with phase exchange
  – Part II – 1.5 Numerical simulations of a volcanic conduit (51 pages)
  –> The relaxation limit of this model is used to obtain a drift-flux system amenable to simplification and comparison with existing simplest models.

Complicated nonlinear PDEs for fluid mechanics for instance may be encountered in various important heterogeneous applications such as volcanology.

The mathematical and numerical study in volcanology
is a real mathematical challenge.

We hope to motivate to have multi-disciplinary interactions
See : http://mathsmonde.math.cnrs.fr/index-en.html

The essential criteria in determining the quality of applied mathematics must be the scientific quality of the questions !!

$\Longrightarrow$ necessity to collaborate with other discipline:

Here with Alain and Marielle!

Writing complicated systems seems to be irresponsable
especially for mathematicians....

But this allows sometimes to propose several new challenges in maths (discrete/continuous)

In vulcanology, this implies new discussions on multi-phase flows and corrections/extensions of what are usually done by geophysicists.

Here we will just make an incursion to vulcanology: Modelization.
For numerics, develop with COMSOL before numerical schemes...
Necessity to collaborate with Gladys !!

Turning around multiphase flows with heterogeneity

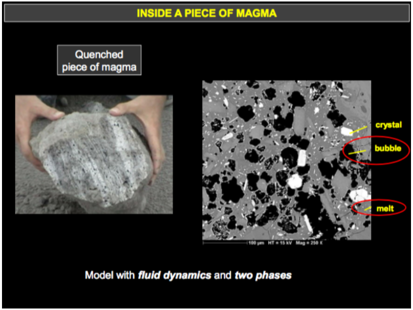
Indicating different mathematical open problems (Theoretical) !

- Weak nonlinear stability
- homogenization procedure:
  From separated monophase to two-phase averaged flows.

What we are talking about today?

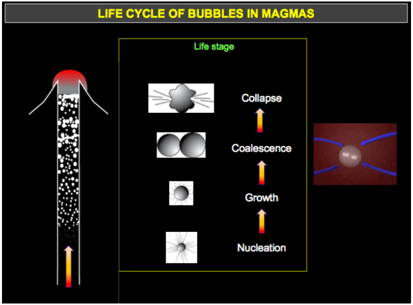
Magma Flowing in a Volcanic Conduit

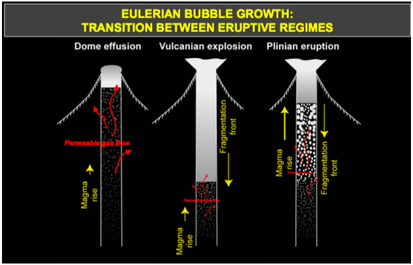
- A multi-phase flow
- A compressible flow
  (obtained from an incompressible/compressible mixture)
- Heat conduction
- Mass exchange
- Bubble growth mechanism



Piece of Magma description

$\Longrightarrow$ two phases (and in fact three phases with crystal)


LIFE CYCLE OF BUBBLES IN MAGMAS
Life stage
Collapse
Coalescence
Growth
Nucleation

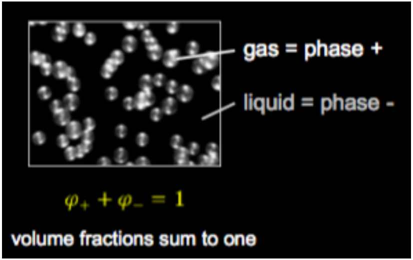

EULERIAN BUBBLE GROWTH:
TRANSITION BETWEEN ERUPTIVE REGIMES
Dome effusion
Vulcanian explosion
Plinian eruption
Permeable gas flow
Magma rise
Fragmentation front
Magma rise
Magma rise
Fragmentation front

- Conservation of mass for the two phases with mass exchanges
- Perfect gas law for gas
- Given density for liquid
- Conservation of mass for the solute component where c is concentration of dissolved gas defined as mass fraction.
- Momentum equations
- Wall stress of the gas negligible
- Liquid wall stress: A non-linear zero order term for diffusion.

It is well understood through Henry's law that the concentration of dissolved gas is dependent on the pressure in the system $c_- = k_h p_+^\nu$ with an empirical constant k_h to choose and a coefficient ν to prescribe (1/2 in our case).

In reality, the growth of bubbles occurs through diffusion of the dissolved gas phase to the bubble surface. This may be real-limiting mechanism for gas exsolution (separation of constituents initially dissolved in a homogeneous phase into a new dispersed phase).

We will need a transport equation on the concentration c_- with a relaxation related to $c_- - K p_+^\nu$.



Two-phase compressible system

$$u_\pm, \qquad \varphi_\pm, \qquad \rho_\pm, \qquad p_\pm.$$

A classical two phase flows:

$$\begin{cases} \varphi_+ + \varphi_- = 1, \\ \partial_t(\varphi_\pm \rho_\pm) + \mathrm{div}(\varphi_\pm \rho_\pm u_\pm) = 0, \\ \partial_t(\alpha_\pm \rho_\pm u_\pm) + \mathrm{div}(\rho_\pm u_\pm \otimes u_\pm) + \varphi_\pm \nabla p_\pm + \mathcal{D}_\pm = F_\pm, \\ p_\pm = c_\pm \rho_\pm \end{cases}$$

where

$$\mathcal{D}_\pm = -2\mathrm{div}(\eta_\pm D(u_\pm)) - \nabla(\lambda_\pm \mathrm{div} u_\pm)$$

$$F_\pm = c_F(u_\pm - u_\mp) + I_\pm$$

and $c_F \in (0, \infty)$ given and $I_\pm$ are interfaces terms.

Algebraic structure

$$p_+ = p_-$$

References modeling :
M. Ishii (1975),
D.A. Drew and S.L. Passman (1998).

See also PhD Thesis by P. Cordesse and references therein.
See M. Massot, V. Giovangigli and co-authors.
See S. Gavryliuk, J.–P. Vila, R. Abgrall, R. Saurel....
for more general two-phase systems.

If $F_{\pm} = 0$:
Non-conservative, non-hyperbolic system if $0 \leq |u_+ - u_-| < c_m$ with

$$c_m^2 = c_-^2 c_+^2 ((\varphi_+ \rho_+)^{1/3} + (\varphi_- \rho_-)^{1/3})^3 / (\varphi_+ \rho_- c_-^2 + \varphi_- \rho_+ c_+^2)$$

In general c_m is large compared to u_+ and u_- and therefore flow belongs to non-hyperbolic region.
H.B. Stewart and B Wendroff (1984).

If $F_\pm = -\pi \nabla \varphi_\pm$ with the bestion term

$$\pi = \delta \frac{\varphi_+ \varphi_- \rho_+ \rho_-}{\varphi_+ \rho^- + \varphi_- \rho_+} (u_+ - u_-)^2$$

with $\delta > 1$ then hyperbolicity for small relative velocity.

More general studies on hyperbolicity with different closures
in D.B., B. Desjardins, J.–M. Ghidaglia, E. Grenier, M. Hillairet.
Hanbook of mathematical analysis in mechanics of viscous fluids 2018.

A classical two phase flows:

$$\begin{cases} \varphi_+ + \varphi_- = 1, \\ \partial_t(\varphi_\pm \rho_\pm) + \mathrm{div}(\varphi_\pm \rho_\pm u_\pm) = 0, \\ \partial_t(\alpha_\pm \rho_\pm u_\pm) + \mathrm{div}(\rho_\pm u_\pm \otimes u_\pm) + \varphi_\pm \nabla p_\pm + \mathcal{D}_\pm = F_\pm, \\ p_\pm = c_\pm(\rho_\pm)^{\gamma_\pm} \end{cases}$$

where

$$\mathcal{D}_\pm = -2\mathrm{div}(\eta_\pm D(u_\pm)) - \nabla(\lambda_\pm \mathrm{div}\, u_\pm)$$

$$F_\pm = c_F(u_\pm - u_\mp) + I_\pm$$

and $c_F \in (0, \infty)$ given and $I_\pm$ are interfaces terms.

PDE structure

$$\partial_t \varphi_\pm + u_\pm \cdot \nabla \varphi_\pm = \varphi_- \varphi_+ \frac{(p_\pm - p_\mp)}{\varepsilon}$$

Monophase system : nonlinear weak stability

Well known models studied in mathematics:
Incompressible Navier-Stokes

$$\left\{ \begin{array}{l} \mathrm{div} u = 0, \\ \rho_*(\partial_t u + \mathrm{div}(u \otimes u)) - \mu \Delta u + \nabla p = \rho_* f, \end{array} \right.$$

where p is the pressure associated to the constraint $\mathrm{div}\, u = 0$ and ν, ρ_* are constants.

J. Leray (1934) : Global weak solutions.

- Anisotropy in the diffusion :
  $2\nu \mathrm{div}(A(t,x)D(u))$ where $D(u) = (\nabla u + {}^t\nabla u)/2$
  with assumptions on the matrix A: OK.

Trick: Aubin-Lions-Simon Lemma: bound on ∇u and $\partial_t u$!

Incompressible non-homogeneous Navier-Stokes

$$\begin{cases} \partial_t \rho + \mathrm{div}(\rho u) = 0, \\ \mathrm{div} u = 0, \\ \partial_t(\rho u) + \mathrm{div}(\rho u \otimes u)) - \mu \Delta u + \nabla p = \rho f, \end{cases}$$

where p is the pressure associated to the constraint $\mathrm{div} u = 0$ and ν is constant.

A. Kazhikov (1977), J. Simon (1986) and then P.–L. Lions (1998): Global weak solutions with vaccum
with $0 \leq \rho_0 \leq C < +\infty$ and $\sqrt{\rho_0} u_0 \in (L^2(\Omega))^d$.

- Anisotropy in the diffusion :
  $2\nu \mathrm{div}(A(t,x)D(u))$ where $D(u) = (\nabla u + {}^t\nabla u)/2$
  with assumptions on the matrix A: OK.
- Viscosity μ depending on ρ: $2\mathrm{div}(\mu(\rho)D(u))$: OK
  with $\mu \in C^1$ and $\mu(s) \geq C > 0$.

$\mu(s) \geq 0$: Open problem !!

Trick: Aubin-Lions-Simon Lemma: bound on ρ and $\partial_t \rho$
$\implies$ ρ is good in time.
But u is good in space so ρu is OK and therefore $\rho u \otimes u$.

Compressible Navier-Stokes

$$\begin{cases} \partial_t \rho + \mathrm{div}(\rho u) = 0, \\ \partial_t(\rho u) + \mathrm{div}(\rho u \otimes u) - \mu \Delta u - (\lambda + \mu)\nabla \mathrm{div}\, u + \nabla p = 0, \end{cases}$$

After $p(\rho) = a\rho^\gamma$ by P.–L. Lions (1993-1998), E. Fereisl (2002), E. Feireisl, A. Novotny and H.Petzeltova (2001), E. Feireisl (2011) (monotone after fixed value $\rho_\star$, Plotnikov and Weigant (2015)

Trick: Commutation between weak limit and a strictly convex function.

Consider $p = p(\rho, \vartheta(t,x))$ with $\vartheta(t,x)$ given.

No assumption such as $\partial_\rho P > 0$!! (application repulsion/attraction).

D.B., P.–E. Jabin, F. Wang (2021) : Global weak solutions à la J. Leray
after D.B. and P–E. Jabin (2018) : Trick: A non-local tool with a policeman !!

Good News: Extension recently by D. B., P.–E. Jabin and F. Wang to Heat-Conducting Navier-Stokes equations with truncated virial pressure law.

The first example of physical pressure law !
Possible to consider anisotropic viscosities with a restriction.

Compressible Navier-Stokes

$$\begin{cases} \partial_t \rho + \mathrm{div}(\rho u) = 0, \\ \partial_t(\rho u) + \mathrm{div}(\rho u \otimes u) - 2\mathrm{div}(\mu(\rho) D(u)) - \nabla(\lambda(\rho)\mathrm{div}\, u) + \nabla p(\rho) = 0, \end{cases}$$

with

$$\lambda(\rho) = 2(\mu'(\rho)\rho - \mu(\rho)).$$

D.B., B. Desjardins (2004): A nonlinear hypo-coercivity property.
D.B., A. Vasseur, C. Yu (2022): Global entropy weak solution.
Remark: Pressure dependent viscosities

Remark:
No result for incompressible Navier-Stokes equation
with pressure dependent viscosity.
$\Longrightarrow$
Same problem for granular flows system with pressure dependent viscosity.

Good News for J. Leray solutions:

- Low regularity based on physical quantities conservations.
- Appropriate to define accurate numerical schemes.

Bad news for J. Leray solutions:
Not enough regularity to follow interfaces evolutions.

Intermediate regularity to be able to follow interfaces:
Weak solutions à la D. Hoff (B. Desjardins): $\rho \in L^\infty(\Omega)$ and $u_0 \in H^1(\Omega)$
More appropriate for averaging procedure:
fast oscillations related to the density (mixture).

The mathematical questions: How to consider more general stress tensors ?
To take into account for instance:

- Maximal packing
- Viscoplastic law
- Pressure dependent viscosities

Weak nonlinear stability? Which mathematical tools to encode compactness ?

A more coupled compactness tool without using what we called effective flux....
In progress.....

$$\begin{cases} \partial_t \rho + \mathrm{div}(\rho u) = 0, \\ \partial_t(\rho u) + \mathrm{div}(\rho u \otimes u) + \nabla p - 2\mathrm{div}\Big(\mu(\rho)(D(u) - \frac{1}{d}\mathrm{div}\, u \mathrm{Id})\Big) = \rho f, \\ \rho u|_{t=0} = m_0, \qquad \rho|_{t=0} = \rho_0. \end{cases}$$

where d space dimension and p is given by solving

$$\mathcal{F}(\mathrm{div}\, u, |D(u) - \frac{1}{d}\mathrm{div}\, u\, \mathrm{Id}|, p, \rho) = 0.$$

Open questions: Conditions on $\mathcal{F}$ for global weak solutions?

- $\mathcal{F}(\mathrm{div}\, u, |D(u) - \frac{1}{d}\mathrm{div}\, u\, \mathrm{Id}|, p, \rho) = \mathrm{div}\, u$
- $\mathcal{F}(\mathrm{div}\, u, |D(u) - \frac{1}{d}\mathrm{div}\, u\, \mathrm{Id}|, p, \rho) = p - p(\rho) + \Big(\lambda(\rho) + \frac{2\mu(\rho)}{d}\Big)\mathrm{div}\, u$

See slides corresponding to L. Chupin's talk: (Roux - Radjai 98):

$$\mathcal{F}(\mathrm{div} u, |D(u) - \frac{1}{d}\mathrm{div u}\,\mathrm{Id}|, p, \rho) = (\mathrm{div} u - 2\sin(\delta)|S(u)|)_{-} + \alpha(\varphi_{\max} - \varphi)\sqrt{p}$$

where

$$S(u) = D(u) - \frac{1}{d}\mathrm{div} u \mathrm{Id}$$

and

$$\tau = \sin(\delta) p \frac{S(u)}{|S(u)|}$$

What is known for multiphase systems concerning weak stability ?

- Algebraic closure
- PDE closures properties

Recent works by

- D.B., B. Desjardins, J.M. Ghidaglia, E. Grenier,
- D.B., P. Mucha, E. Zatorska
- D.B., J. Li, X. Huang
- C. Michoski, A. Vasseur,
- A. Novotny- M. Pokorny,
- A. Novotny,
- A. Vasseur–Wen–C. Yu.

Originalities

- Pressure depending on two parameters
- Viscosity depending on fraction
- Non-conservative terms

From mono-phase with interfaces

to

Two-phase averaged systems: homogenization procedure

Interesting mathematical problems :
1) How to obtain multiphase macroscopic system from a mesoscopic description with interfaces?
2) What is ε from a physical point of view?

This has been mathematically justified for joint velocity by
D.B. and M. Hillairet: Same pressure laws.
D.B., C. Burtea, F. Lagoutière: Different pressure laws (color function).
Recently work with heat equation with zero conduction coefficient:
SEE POSTER P. GONIN-JOUBERT AND DISCUSS WITH HIM
Collaboration with C. Burtea, F. Lagoutière and D.B.

Observation: ε is linked to the shear and bulk viscosities.

Hint: Start with oscillating initial density which provides a linear combination of appropriate dirac masses and characterize the Young measures family for all time.
$\Longrightarrow$ multi-phase equation with PDE closure justification !!

The mathematical system with Gladys, Marielle and Alain

Two phase compressible-incompressible flows in volcanology

$$\begin{cases} \varphi_+ + \varphi_- = 1, \\ \partial_t(\varphi_\pm \rho_\pm) + \mathrm{div}(\varphi_\pm \rho_\pm u_\pm) = 0, \\ \partial_t(\alpha_\pm \rho_\pm u_\pm) + \mathrm{div}(\rho_\pm u_\pm \otimes u_\pm) + \nabla(\varphi_\pm p_\pm) + \mathcal{D}_\pm - \varphi_\pm \rho_\pm g = \pm F \mp p_{\mathrm{int}} \nabla \varphi_\pm, \\ p_+ = c_+ \rho_+, \qquad \rho_- \text{ given} \end{cases}$$

where

$$\mathcal{D}_\pm = -2\mathrm{div}(\eta_\pm D(u_\pm)) - \nabla(\lambda_\pm \mathrm{div}\, u_\pm)$$

$$F = K_d \varphi_+ \varphi_- (u_+ - u_-)$$

and $K_d \in (0, \infty)$ given.

PDE closure

$$\partial_t \varphi_+ + u_{\mathrm{int}} \cdot \nabla \varphi_+ = \varphi_+ \varphi_- \frac{p_+ - p_-}{\varepsilon}$$

Choice:

$$p_{\mathrm{int}} = p_+ \text{ and } u_{\mathrm{int}} = u_-.$$

Guillemaud (2007)
Gallouët, Hérard and Seguin (2004)

Two bubble growth mechanisms:

$$\partial_t \varphi_+ + \mathrm{div}(\varphi_+ u_-) = \varphi_+ \varphi_- \frac{p_+ - p_-}{\varepsilon} + \varphi_+^2 \mathrm{div}\, u_-$$

and by mass addition

$$\begin{cases} \varphi_+ + \varphi_- = 1, \\ \partial_t(\varphi_\pm \rho_\pm) + \mathrm{div}(\varphi_\pm \rho_\pm u_\pm) = \pm R, \\ \partial_t(\alpha_\pm \rho_\pm u_\pm) + \mathrm{div}(\rho_\pm u_\pm \otimes u_\pm) + \nabla(\varphi_\pm p_\pm) \\ \qquad\qquad + \mathcal{D}_\pm \mp u_- R - \varphi_\pm \rho_\pm g = \pm F \mp p_+ \nabla \varphi_\pm, \\ p_+ = c_+ \rho_+, \qquad \rho_- \text{ given} \end{cases}$$

Remark: Equations on φ_+ and $\varphi_- \rho_-$
$\Longrightarrow$ constraints on $p_+ - p_-$ related to $\mathrm{div}\, u_-$

Open problem: Mathematical low mach numbers limits with two phases.

$$R = D\, c(\varphi, n, \rho_-)(c_- - k_h\sqrt{p_+}).$$

The equation on c_- reads

$$\partial_t(\varphi_-\rho_-c_-) + \mathrm{div}(\varphi_-\rho_-u_-c_-) = -R$$

To complete :

- Need equations on T_+, T_- at least one temperature.
- Need to introduce the number of bubble in the liquid volume $N = n/\varphi_-$ where n number of bubble per total volume.

$\Longrightarrow$ PDE for N.
$\Longrightarrow$ terms with N in PDE for φ_+.

Appropriate temperature equations
provides an important energy dissipation property

From a mathematical point of view:

Several interesting singular behavior to some equilibrium !!

$$c_- = k_h \sqrt{p_+}$$

and

$$p_+ = p_-$$

More general for Navier-Stokes with temperature without heat conduction.

Constraints and necessity:

- Use COMSOL before a numerical code......
- Add the coupling with the unsaturated porous lateral wall...
- Dome formation and evolution on the top....
- Three phases flows....

Numerical and mathematical difficulties:

- Need to discuss boundary conditions :
  No trivial boundary conditions in conduit
- Need to simplify the difficulties but not too much
  to keep the physics corresponding to vulcano
- One dimensional case first for numerics.

Drift-Flux system and relaxation limits

Drift flux system based on the choice other unkowns

$$\rho = \varphi_+ \rho_+ + \varphi_- \rho_-$$

$$\rho Y = \varphi_+ \rho_+$$

$$u = Y u_+ + (1 - Y) u_-$$

$$w = u_g - u_l$$

$$p = \varphi_+ p_+ + \varphi_- p_-$$

$$q = p_+ - p_-$$

Steps:

- Write an equation on ρ, Y, u and w.
- Simplified using adimensional numbers

Basic two-phase system

$$\partial_t \rho + \mathrm{div}(\rho u) = 0$$

$$\partial_t(\rho Y) + \mathrm{div}(\rho Y u + \rho Y(1-Y)w) = 0$$

$$\partial_t(\rho u) + \mathrm{div}(\rho u \otimes u + p\mathrm{Id} + \rho Y(1-Y)w \otimes w) = \rho g$$

and more importantly

$$\left\{ \begin{array}{l} \partial_t w + \mathrm{div}[(1-Y)w \otimes w] + u \cdot \nabla w + w \cdot \nabla u - Yw \cdot \nabla w - (1-Y)w\mathrm{div} w \\ +K_d w = \frac{1}{\rho Y(1-Y)}[(Y-\varphi_+)\nabla p - \varphi_+ \nabla(\varphi_- q)] \end{array} \right.$$

It is interesting to see that in several papers, multi-d has been derived by extrapolation from one-d................ which provides problem sometimes since in one-d $\mathrm{div} = \nabla$......

For instance they write

$$\mathrm{div}[(\frac{1}{2} - Y)w \otimes w]$$

instead of

$$\mathrm{div}[(1 - Y)w \otimes w] - Yw \cdot \nabla w - (1 - Y)w\mathrm{div}w$$

in the equation of w....

In one-d this is true but not in multi-d....... The true terms are

$$\mathrm{div}[(1-Y)w\otimes w] - Yw\cdot\nabla w - (1-Y)w\,\mathrm{div}w$$

and not

$$\mathrm{div}[(\frac{1}{2}-Y)w\otimes w]$$

Drift Flux

$$\partial_t \rho + \mathrm{div}(\rho u) = 0$$
$$\partial_t(\rho Y) + \mathrm{div}(\rho Y u + \rho Y(1 - Y) w) = 0$$
$$\partial_t(\rho u) + \mathrm{div}(\rho u \otimes u + p\mathrm{Id}) = \rho g$$

with

$$w = \frac{1}{K_d} \varphi_+ \varphi_- \frac{\rho_g(p) - \rho_l}{\rho} \nabla p.$$

The bad terms do not inter in the well known drift flux limit.

Drift flux system derivation for our problem is more complicated due to the specification of the volcanology modelization.

Some extra terms could be important.

Temperature ! size of w and more specifically Yw !!

Open mathematical problem: Mathematical justification of the drift-flux system

Future possible collaborations for Alain and Marielle:

- Time to adapt CALIF3S to vulcanology purposes.....
  Discussions with W. Boscheri + K. Saleh.
- Time to include evolutions of bubbles....
  Discussions with C. Lacave
- Time to include chemical process.....
  Discussions with M. Kessar

Time to discuss with other mathematical expert on multiphase flows...

Thank you for your attention!

Do not hesitate to exchange with me by email: didier.bresch@univ-smb.fr