Asymptotic Analysis of Electrocardiology Modeling after Pulsed Field Ablation

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In this work, we focus on the mathematical study of pulsed electric field ablation (PFA), an innovative cardiac ablation technique for the treatment of cardiac arrhythmias. In particular, we would like to compare it with radiofrequency ablation (RFA), a thermal ablation that is currently the most commonly used technique. Whereas RFA is known to result in coagulation necrosis with complete loss of cellular and vascular architecture, leaving a scar of fibrotic tissue, PFA takes advantage of irreversible electroporation – a complex cell death phenomenon that occurs when a biological tissue is subjected to very intense electric pulses – and it is known to destroy mainly cardiomyocytes while preserving the tissue scaffold.
This work aims to modify the classical bidomain model [2] – which describes the propagation of intracellular and extracellular potentials in the heart – to introduce a region ablated by RFA or PFA. Both types of ablation involve isolation of a pathological area, but we describe them differently by using appropriate transmission conditions at the interface between the ablated area and the not-ablated area. In the case of RFA, we assume that both intracellular and extracellular potentials are affected, resulting in Kedem-Katchalsky-type conditions at the interface. In contrast, in the case of PFA, we study the static bidomain model and we assume that the thickness of the electroporated (EP) region is small compared with the whole domain and proportional to a paramater ε. Moreover, we assume that within the EP region the intracellular conductivity scales with a factor ε^2. We provide a formal asymptotic analysis at any order by considering an asymptotic expansion of the intracellular and extracellular potentials both outside and inside the EP area. This allows us to derive transmission conditions at the interface for PFA at any order, that read as non-homogeneous boundary conditions for the jump of the extra-cellular potential and its normal derivative, and as Neumann conditions for the intracellular potentials. Moreover, we give a proof of the asymptotic expansion by deriving estimates of H^1- and L^2- norms of the errors of an expansion with a given number N of terms. The asymptotic expansion was validated by numerical convergence tests. In particular, zero- and first-order expansions were compared and errors were computed for different values of ε tending to zero.
Finally, we propose physical simulations [1] in the context of atrial fibrillation (AF). We consider the isolation of one pulmonary vein of a synthetic geometry of the left atrium. In the case of PFA, we consider transmission conditions deduced from the zero-order asymptotic analysis of the static case. Numerical simulations of AF and the long-term effects of RFA and PFA show that both models lead to isolation of the pulmonary vein. Our modeling also enables to propose a numerical explanation for the higher rate of fibrillation recurrence after RFA compared with PFA.

[1] S. Nati Poltri, G. Caluori, P. Jaïs, A. Collin, C. Poignard. *Electrocardiology modeling after catheter ablations for atrial fibrillation.* In *International Conference on Functional Imaging and Modeling of the Heart.* Springer, 2023.

[2] L. Tung. *A bi-domain model for describing ischemic myocardial dc potentials.* Ph.D. thesis, Massachusetts Institute of Technology, 1978.

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