Convergence rate of entropy-regularized multi-marginal optimal transport costs

Luca NENNA, LMO - Orsay **Paul PEGON**, CEREMADE - Paris

We investigate the convergence rate of multi-marginal optimal transport costs that are regularized with the Boltzmann-Shannon entropy, as the noise parameter ε tends to 0. We establish lower and upper bounds on the difference with the unregularized cost of the form $C\varepsilon \log(1/\varepsilon) + O(\varepsilon)$ for some explicit dimensional constants C depending on the marginals and on the ground cost, but not on the optimal transport plans themselves. Upper bounds are obtained for Lipschitz costs or semi-concave costs (for a finer estimate), and lower bounds for C^2 costs satisfying some signature condition on the mixed second derivatives that may include degenerate costs, thus generalizing results previously obtained with Carlier and Tamanini [1], and by Eckstein and Nutz [2]. We obtain in particular matching bounds in some typical situations where the optimal plan is deterministic, like in the case of Wasserstein barycenters. This is a joint work with Luca Nenna [3].

[1] G. Carlier, P. Pegon, L. Tamanini. *Convergence rate of general entropic optimal transport costs.* Calculus of Variations and Partial Differential Equations, **62(4)**, 116, 2023. doi :10.1007/s00526-023-02455-0.

[2] S. Eckstein, M. Nutz. *Convergence Rates for Regularized Optimal Transport via Quantization.* Mathematics of Operations Research, 2023. doi :10.1287/moor.2022.0245.

[3] L. Nenna, P. Pegon. *Convergence rate of entropy-regularized multi-marginal optimal transport costs.* Canadian Journal of Mathematics, pp. 1–22, 2024. doi :10.4153/S0008414X24000257.

Contact : pegon@ceremade.dauphine.fr