Hybrid AI based reduction of the shallow water system for flood plain simulations

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Reduced Order Models (ROMs) have been widely used to efficiently solve large-scale problems in many fields including computational fluid dynamics (CFD) [4]. ROMs techniques allow to replace the expansive Full Order Model (FOM), by a ROM that captures the essential features of the system while significantly reducing the computational cost.
In this work, we elaborate on and evaluate a new reduced basis method referred to as EA-POD-NN [2] for model reduction of the shallow water equations, which combines Proper Orthogonal Decomposition (POD) with Artificial Neural Networks (ANNs). The method begins with the POD technique to construct reduced bases from high-resolution solutions and then utilizes deep ANNs to learn associated coefficients in the reduced bases as it has been initially done in [3]. The method follows an offline-online strategy : the POD reduced basis, along with the training of the ANNs, are performed in an offline stage, enabling the surrogate model to be used for real-time predictions. The process is non-intrusive, as it does not require opening the high-resolution model code.
Here, the proposed method additionally takes into account the POD-based projection error, enabling the attainment of higher accuracy while preserving a limited number of POD modes, even in the delicate situation of non-linear advection-dominated flows. The approach allows for the limitation of the output dimension of the ANNs, thus enabling the use of smaller ANNs with fewer parameters, therefore a better interpretability and potentially smaller datasets. We further compare its performance with the original method, POD-NN derived in [3].

The developed method is evaluated on a real-world test case aimed at simulating inundation of the Aude river in Southern France, and relies on the open-source computational code DassFlow [1]. Results demonstrate that the proposed EA-POD-NN method provides satisfying accuracy for hydraulic variables such as water elevation and discharge. Overall, the method is promising for performing real-time simulations of floodplain dynamics in particular (but not only), offering a cost-effective approach for ROM of shallow water equations.

[1] *Dassflow (data assimilation for free surface flows). open source computational software.* Project-team MathHydroNum (IMT/INSA Toulouse - INRAe Aix-en-Provence - ICUBE Strasbourg). https ://github.com/DassHydro.

[2] M. Allabou, R. Bouclier, P. A. Garambois, J. Monnier. *Reduction of the shallow water system by an error aware pod-neural network method : Application to floodplain dynamics.* submitted (revised version).

[3] J. S. Hesthaven, S. Ubbiali. *Non-intrusive reduced order modeling of nonlinear problems using neural networks.* Journal of Computational Physics, **363**, 55–78, 2018.

[4] A. Quarteroni, A. Manzoni, F. Negri. *Reduced basis methods for partial differential equations : an introduction*, vol. 92. Springer, 2015.

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