Computing the weak optimal transport with moment constraints via entropic regularization

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The weak optimal transport introduced in [7] is an extension of optimal transport which takes the following form

$$\inf_{\pi \in \Pi(\mu,\nu)} \int c_x(\pi_x) d\mu$$

where $\pi = \mu \otimes \pi_x$ and c_x is a function defined over the probability measures. This formulation includes the entropic optimal transport and has multiple applications [2] such as the martingale optimal transport [3], vector quantile regression [4]. Duality attainment results have been obtained in the non entropic case [7][1]. We will show that dual attainment holds for costs of the form $c_x(p) = c_x^0(p) + 1_{\int f(y)dp(y)=0} + \epsilon H(p \mid \nu)$ where c_x^0 is a Lipschitz (uniformly in x) for TV norm convex function, f is a vector valued function and $\epsilon \geq 0$. Moreover we derive regularity (at least L^∞) for the dual potentials. Which in turns grant quantitative stability result in the marginal and in ϵ by using a modified version of the block approximation [5]. Finally the convergence of the numerical scheme is proven and applications such as the Brenier-Strassen interpolation [6] are computed.

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