Numerical methods for yield-stress fluids governed by integral constitutive laws

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Many phenomenological constitutive models for yield-stress fluids are formulated as a nonsmooth algebraic or differential equation, which links a stress tensor to the instantaneous rate of strain. This includes Bingham fluids, Herschel-Bulkley fluids and elastoviscoplastic extensions of these models, among others. State-of-the-art numerical methods for simulating flows of such fluids predominantly fall into two groups : algorithms based on nonsmooth convex optimisation and algorithms based on Newton's method.

In this talk, we consider microscopic material descriptions based on the mode-coupling theory for fluids, which at the molecular scale are composed of hard spheres. These constitutive laws are mathematically formulated as an integral equation for a stress tensor in terms of the fluid's deformation history. The strength of such, admittedly, rather complex microscopic models is that they can not only describe, but even physically explain non-Newtonian phenomena such as yielding from first principles. Their algorithmic treatment is, naturally, significantly more involved.

To solve the coupled system of Navier-Stokes equations and integral constitutive laws, we discuss a deformation fields method that keeps track of deformations at past times. We present a structure-preserving discretisation and iterative scheme for the full problem. Interestingly, classical phenomenological constitutive models can be recovered as limiting cases of the more complex microscopic description. This additionally allows us to study new algorithms that result for simple viscoplastic fluids after taking this limit.

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