A finite volume scheme for vanishing viscosity solutions on a star-shaped graph under Kedem-Katchalski transmission conditions at the node

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We consider a family of conservation laws defined on a star-shaped oriented graph Γ consisting of m incoming edges $\Omega_i, i = 1\ldots, m$ and n outgoing edges $\Omega_j, j = m+1,\ldots,m+n$ joining at a sigle node. We assume that the edges are parametrized by $x \in (-\infty, 0]$ for $k = 1,\ldots,m$ and $x \in [0,+\infty)$ if $k = m+1,\ldots,m+n$. On each edge Ω_k, we introduce a scalar conservation law which describes the evolution of a density ρ_k, $\partial_t\rho_k + \partial_x f_k(\rho_k) = 0$, where f_k is bell-shaped. An admissible solution is a vector $\vec{\rho} = (\rho_1,\ldots,\rho_{m+n})$ such that ρ_k is an entropy solution in the interior of Ω_k for all $t > 0$, and $\vec{\rho}$ satisfy a given transmission condition at the node. Different type of conditions at the node exists in the litterature depending on the physical situation we wish to model. In [1], the authors proposed a numerical scheme for solutions obtained as limits of vanishing viscosity approximation with a condition of density continuity at the node (at the parabolic level). Two implementations are proposed in [2] and [3]. The main focus of this talk will be to approximate solutions of the vanishing viscosity problem supplemented with a different kind of conditions at the node :

$$\begin{cases} f_i(\rho_i(t,0^-) - \epsilon\partial_x\rho_i(t,0^-) = \sum_{j=m+1}^{m+n} c_{ij}(\rho_i(t,0^-),\rho_j(t,0^+)), & t>0, i=1,\ldots,m, \\ -f_j(\rho_j(t,0^-) + \epsilon\partial_x\rho_j(t,0^+) = \sum_{i=1}^{m} c_{ij}(\rho_j(t,0^+),\rho_i(t,0^-)), & t>0, j=m+1,\ldots,m+n, \end{cases}$$

for $\epsilon > 0$, for given $c_{ij} \geq 0, i = 1,\ldots,m, j = m+1,\ldots,m+n$ Lipschitz continuous and $c_{ij}(.,\rho_j)$ is increasing and $c_{ij}(\rho_i,.)$ is decreasing. In the case $m = 1, n = 2$, we developped and implement a finite volume scheme which is inspired from [1] to approximate the solution of the problem, then we apply it to a simple model for pedestrian traffic.

[1] B. P. Andreianov, G. M. Coclite, C. Donadello. *Well-posedness for vanishing viscosity solutions of scalar conservation laws on a network.* Discrete and Continuous Dynamical Systems, **37(11)**, 5913–5942, 2017. doi :10.3934/dcds.2017257.

[2] S. F. Pellegrino. *On the implementation of a finite volumes scheme with monotone transmission conditions for scalar conservation laws on a star-shaped network.* Applied Numerical Mathematics, **155**, 2019. doi :10.1016/j.apnum.2019.09.011.

[3] J. D. Towers. *An explicit finite volume algorithm for vanishing viscosity solutions on a network.* Networks and Heterogeneous Media, **17(1)**, 1–13, 2022. doi :10.3934/nhm.2021021.

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