Numerical investigations of Lugiato-Lefever equation for Kerr combs generation in Fabry-Perot resonators

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Kerr frequency combs refer to a laser source whose spectrum consists of a series of discrete, equally spaced frequency lines, which are generated in an optical cavity from a continuous wave pump laser by Kerr nonlinearity. The coherent conversion of the pump laser to a frequency comb has been obtained in a variety of optical resonators such as whispering-gallery mode ring resonators, fiber ring resonators and more recently Fabry-Perot (FP) resonators, our main concern here [2]. FP resonators are formed by an optical fiber (centimeters long) bounded at each end by a multi-layer dielectric mirror.

A model to analyze light-wave propagation in ring as well as FP resonators in the context of Kerr comb generation relies on the Lugiato–Lefever equation (LLE). It is a nonlinear Schrödinger equation with damping, detuning, and driving force. With unknown ψ related to the slowly varying electric field envelope, the Lugiato–Lefever equation reads

$$\frac{\partial\psi}{\partial t}(\theta,t) = -\mathrm{i}\frac{\beta}{2}\,\frac{\partial^2\psi}{\partial\theta^2}(\theta,t) - (1+\mathrm{i}\alpha)\,\psi(\theta,t) + \mathrm{i}\,\psi(\theta,t)\left(|\psi(\theta,t)|^2 + \frac{\sigma}{\pi}\int_{-\pi}^{\pi}|\psi(\zeta,t)|^2\,\mathrm{d}\zeta\right) + F \qquad (1)$$

where $\theta \in]-\pi,\pi[$ is a normalized space variable introduced to cover in a unified way the above mentioned resonator types, t is a local time and the parameter σ has value either 0 for ring resonators or 1 for FP resonators. The constant unit-less parameters in (1) are $F > 0$ the amplitude of the laser pump, β the cavity group velocity dispersion (GVD) parameter and α the cavity phase detuning of the laser pump. The LLE (1) is to be considered together with periodic boundary and initial conditions.

The LLE (1) can be very efficiently solved numerically by using a Split-Step method but long-time solutions are strongly sensitive to the initial condition. Fortunately, since Kerr frequency comb are related to LLE stationary solutions, an alternative numerical approach to investigate frequency combs is to solve the stationary equation deduced from (1) using e.g. a collocation method. However, we have to face up to a new difficulty : most of the initializations for the collocation method lead to trivial constant solutions that are irrelevant from a physical point of view. This led us to refer to the bifurcation theory to investigate spatially periodic solutions to (1) bifurcating from the explicitly known branch of trivial constant solutions as devised in [1] for $\sigma = 0$.

Based on this analysis, we are using a pseudo-arc-length numerical continuation method from the onsets of bifurcations branches on the trivial branch of solutions, with either F or α as bifurcation parameter. We obtain a very rich bifurcation picture which allows an extensive account of various possible types of solutions to the stationary LLE. Moreover, in a complementary way, we can obtain close to their onsets an accurate knowledge of the bifurcations branches through asymptotic expansions.

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[1] R. Mandel, W. Reichel. *A priori bounds and global bifurcation results for frequency combs modeled by the Lugiato-Lefever equation.* SIAM J. Appl. Math., **77(1)**, 315–345, 2017.

[2] J. Musgrave, S.-W. Huang, M. Nie. *Microcombs in fiber Fabry–Pérot cavities.* APL Photonics, **8(12)**, 121101, 2023.

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