A consistent depth-averaged model for granular flow with $\mu(I)$ rheology

Émile DELÉAGE, I2M - Marseille

We present a depth-averaged model for an incompressible granular flow down an inclined plane with $\mu(I)$ rheology. The first two unknowns of the model derived are the height and the mean horizontal velocity. The shear is also taken into account via a third variable called enstrophy. Mathematically, the system obtained is a hyperbolic system of three conservation laws, with an additional equation for the total energy. The system is obtained from an asymptotic expansion in powers of the shallow water parameter ϵ. This method ensures that the depth-averaged model is consistant with the bulk rheology. As a consequence, the stability threshold above which roll waves appear is the same for the depth-averaged model and for the bulk equations. In addition to the first order hyperbolic system of equations, we also present a higher-order version that contains diffusive terms. The spatial growing rate and the phase velocity of the second version are in good agreement with the experimental data of Forterre and Pouliquen, (Journal of Fluid Mechanics, 486 :21-50, 2003). Finally, the hyperbolicity of the equations enables to use well-known and stable numerical solvers. As an example, we used our model to perform simulations of granular roll waves.

Contact : emile.deleage@univ-amu.fr