Annealed quantitative estimates for the quadratic 2D-discrete random matching problem

Nicolas CLOZEAU, ISTA - Vienna **Francesco MATTESINI**, MPI - Leipzig

We study the following quadratic matching problem on the flat torus $\mathbb{T}^2$

$$\min_{\pi\in\Pi_{nm}} \sum_{i=1}^{n}\sum_{j=1}^{m} \pi_{ij}|X_i - Y_j|^2 \quad \text{with } \Pi_{nm} := \Big\{\pi \in [0,1]^{n\times m} \Big| \sum_{i=1}^{n} \pi_{ij} = \frac{1}{m} \text{ and } \sum_{j=1}^{m} \pi_{ij} = \frac{1}{n}\Big\}, \tag{1}$$

between two families of random points $\{X_k\}_{1\leq k\leq n}$ and $\{Y_k\}_{1\leq k\leq m}$ (with $m \geq n$) that may possess correlations and that are identically distributed according to a density ρ that satisfies

$$\lambda \leq \rho \leq \Lambda \quad \text{for } \lambda, \Lambda > 0.$$

We are interested in approximating the minimizers of (1) in the regime $n, m \uparrow \infty$. Using the point of view of optimal transport, I will present a PDE approach for approximating the minimizers of (1) based on a formal linearisation of the Monge-Ampère equation ; and explain how that can be made rigorous in a quantitative way. This is based on the publication [1].

[1] N. Clozeau, F. Mattesini. *Annealed quantitative estimates for the quadratic 2d-discrete random matching problem.* Probability Theory and Related Fields, pp. 1–57, 2024.

Contact : `nicolas.clozeau@ist.ac.at`