Cell-by-Cell Model of Cardiac Electrophysiology: Finite Volume Approaches and Numerical Analysis

Zeina CHEHADE, IMB, UMR 5251, IHU Liryc, Inria Carmen - F-33400 Talence

Yves COUDIÈRE, IMB, UMR 5251, IHU Liryc, Inria Carmen - F-33400 Talence

Recently, there has been a growing interest in the extracellular-membrane-intracellular cardiac model (EMI), which accurately represents the discrete cellular structure of the myocardium. This model consists of piecewise electrostatic equations coupled through non-standard time dependent transmission conditions on the myocyte membranes (see [3]). For the aforementioned model, the finite volume methods (FVM) turn out to be good candidates to provide the required accuracy and robustness. As part of the European project MICROCARD, we investigated the convergence of the classical FVM, Two-Point Flux Approximation (TPFA), as defined in [1], with Backward-Differentiation Formula (BDF) and Rush-Larsen (RL) [2] methods for time integration. Through the finite volume formulation, a discrete Dirichlet-to-Neumann operator can be introduced to map the EMI model to an evolution equation on the membranes only. Consequently, FVM naturally yield a large sparse linear system similar to that of Finite Element Methods, or a much smaller one exclusively for the membrane unknowns, as observed in Boundary Element Methods. The convergence analysis using an Implicit-Explicit Euler time-stepping approach, was carried out on the electrical potential in the intra- and extra-cellular subdomains, and on the transmembrane voltages at cell interfaces, using a discrete H^1-like semi norm, L^2 and L^∞ norms in space, along with L^2 and L^∞ norms in time [1].
Afterward, we conducted numerical studies for these errors, with the implicit-explicit Euler method, as well as BDF2 and RL2, using manufactured solutions in two-dimensional cases. Moreover, we computed some illustrative electrical wave propagation. A challenge encountered lies in constructing admissible meshes. To address this, we used software tools provided by some project partners.
For regular manufactured solutions, we observed convergence at order 1 in the discrete H^1 semi-norm, and at order 2 in the L^2 norm on the electrical potential, as expected for FVM in this context. We also noted convergence at order 2 for the membrane voltage in the L^2 norm, which remains unexplained. In our tests, the number of nonzero entries in sparse and dense linear systems were very similar, and the solutions obtained by both approaches are equal up to numerical precision.
The extension of TPFA method to higher dimensions proves challenging due to the admissibility constraint. Consequently, we intend to explore more flexible FVM, such as the hybrid finite volume methods.
This work is part of the MICROCARD project. This project has received funding from the European High-Performance Computing Joint Undertaking EuroHPC (JU) under the grant agreement No 955495.

[1] Z. Chehade, Y. Coudière. *The two-point finite volume scheme for the microscopic bidomain model of electrocardiology.* In E. Franck, J. Fuhrmann, V. Michel-Dansac, L. Navoret, eds., *Finite Volumes for Complex Applications X—Volume 1, Elliptic and Parabolic Problems*, pp. 235–243. Springer Nature Switzerland, Cham, 2023.

[2] M. E. Marsh, S. T. Ziaratgahi, R. J. Spiteri. *The secrets to the success of the rush–larsen method and its generalizations.* IEEE Transactions on Biomedical Engineering, **59**, 2506–2515, 2012.

[3] A. Tveito, K. H. Jæger, M. Kuchta, K.-A. Mardal, M. E. Rognes. *A cell-based framework for numerical modeling of electrical conduction in cardiac tissue.* Frontiers in Physics, **5**, 2017. doi : 10.3389/fphy.2017.00048.

Contact : `zeina.chehade@u-bordeaux.fr`