Positivity-preserving DDFV scheme for compressible two-phase Darcy flow in porous media

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In this work, we are interested in computing good approximations of solutions to the compressible two-phase Darcy flow in porous media model. This classic model describes the flow of a phase of gas g and one of water w, through a porous media Ω, in finite time t_f (one denotes $Q_{t_f} = \Omega \times (0, t_f)$). These phases are considered immiscible and saturing. The model consists of the following system of two coupled and degenerate parabolic equations

$$\begin{cases} \phi \partial_t(\rho_\alpha s_\alpha) + \operatorname{div}(\rho_\alpha \mathbf{V}_\alpha) + \rho_\alpha q^\alpha = 0 \quad \text{in} \quad Q_{t_f}, \quad \forall \alpha \in \{g, w\}, \\ \mathbf{V}_\alpha = -M_\alpha(s_\alpha)\Lambda(\nabla p_\alpha - \rho_\alpha \mathbf{g}), \quad \forall \alpha \in \{g, w\}, \\ s_w + s_g = 1 \quad \text{and} \quad p_c(s_g) = p_g - p_w. \end{cases} \tag{1}$$

For each phase α, s_α is its saturation, p_α its pressure, $\rho_\alpha(p_\alpha)$ its density, and $M_\alpha(s_\alpha)$ its degenerate mobility (i.e. $M_\alpha(s_\alpha = 0) = 0$). The porous medium is described by its porosity $\phi(x)$, its permeability tensor $\Lambda(x)$ and its capillary pressure law p_c. We complete our system with initial conditions, Dirichlet, and Neumann boundary conditions.
The Discrete Duality Finite Volume (DDFV) framework uses a primal mesh $\mathfrak{M}$, a dual mesh $\overline{\mathfrak{M}^*}$ and diamond one $\mathfrak{D}$ to approach gradients constantly by diamond on quite general meshes $\mathfrak{M}$, and treating anisotropic permeability tensor. We propose a positivity-preserving DDFV scheme for (1) in [2]. The originality of our approach lies in the way we approximate the fluxes through the interfaces between cells. We use two different upwind approximations, in the normal direction we use an upwinding approximation of the phase mobilities with respect to the discrete gradient of their pressure, and in the tangential direction, the mobilities are split into two parts : an upwind mobility, and a minimal approximation of the mobilities.
The solutions of such a scheme preserve the physical bounds of the saturations $0 \leq s_{\alpha,\mathcal{T},\delta t} \leq 1$. In addition, one obtains energy estimates on the global pressure and a capillary term, needed because of the degeneracy of the mobilities. It writes with a constant C depending on the regularity of the mesh, the initial conditions, the physical parameters of the medium, but independent of the time step, and the refinement of the mesh, as follows :

$$\left\| \nabla^{\mathfrak{D}} p_{\mathcal{T},\delta t} \right\|_2^2 + \left\| \nabla^{\mathfrak{D}} \xi_{\mathcal{T},\delta t} \right\|_2^2 + \frac{\gamma}{h_{\mathfrak{D}}^{\epsilon}} \left(\left\| \xi_{\mathfrak{M},\delta t} - \xi_{\overline{\mathfrak{M}^*},\delta t} \right\|_{L^2(Q_{t_f})}^2 + \left\| p_{\mathfrak{M},\delta t} - p_{\overline{\mathfrak{M}^*},\delta t} \right\|_{L^2(Q_{t_f})}^2 \right) \leq C.$$

The existence of solutions to this scheme is a delicate point, we give a proof in [1] using regularization of such a scheme. Moreover, one has the convergence toward a weak solution of model (1), up to compacity arguments and using the Lions-Aubin-Simon theorem. Finally, we perform numerical tests to show the efficiency of our method.

[1] T. Crozon. *Existence of solutions to numerical schemes using regularization : application to two-phase flow schemes.* Submitted, 2024.

[2] T. Crozon, E.-H. Quenjel, M. Saad. *Positivity-preserving ddfv scheme for compressible two-phase flow in porous media.* Computers and Mathematics with Applications, 2024.

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