Geometrical considerations on some potential problems

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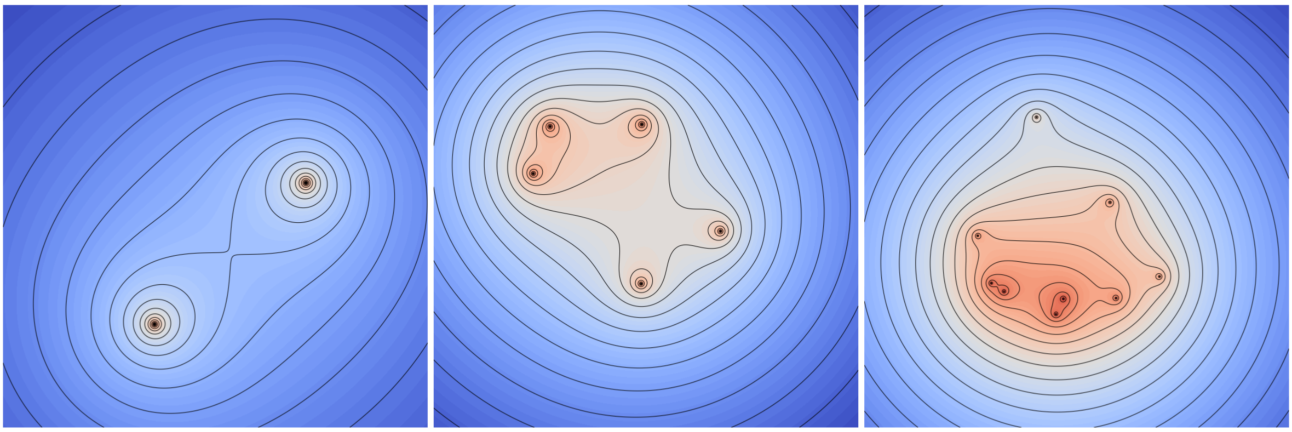
Classical potential problems deriving from the gradient of a scalar field Φ are a well-known class of mathematical problems with applications to electromagnetics, thermodynamics, and fluid flows. Finding Φ, defined on a domain $\Omega \in \mathbb{R}^n$, thus consists in solving the following differential system :

$$\overrightarrow{\nabla} \cdot \left(\kappa \overrightarrow{\nabla} \Phi\right) = S \text{ in } \Omega \quad \text{and with } \alpha\Phi + \beta \frac{\partial \Phi}{\partial \overrightarrow{n}} = \gamma \quad \text{on domain boundaries.}$$

Numerical resolution for this class of problems has been extensively studied and discussed throughout the years in the literature. In particular, methods like the *Finite Elements* (FE), the *Finite Volume Method* (FVM) and the *Boundary Element Method* (BEM) have already provided the industry with software that may solve virtually any configuration especially the most complex ones with strong non-linearities.

However, the constant development of increasingly sophisticated algorithms, combined with readily available computing power, has made the search for analytical solutions to most industrial problems unnecessary. This contribution aims to highlight some analytical solutions that are rooted in the acknowledgment of historically remarkable curves. Such notable curves include the *generalized Cassini curves* [1], which lays out the the typical electrostatic (or temperature) field distribution created by localized sources within a plane (see Figure 1). Interestingly, for the 2-poles case, the Cassini curves can also be described as sections of a torus, prompting inquiry into whether sheer geometric considerations can yield an appropriate parametrized surface for the generalized N-poles case.

We therefore propose in this talk to address this question and to further explore links between geometry and potential problems, then compare the outcomes of this study to reference test cases.



(a) 2 poles (b) 5 poles (c) 9 poles

FIGURE 1 – Generalized Cassini curves for a random arrangement of poles in the xy-plane

Références

[1] E. L. Morgan. *An application of the theory of the complex variable to the mapping of some potential functions.* Journal of the Franklin Institute, **243(4)**, 309–322, 1947. doi : https ://doi.org/10.1016/0016-0032(47)90149-X.

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