Learning non-canonical Hamiltonian dynamics

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We want to study the application of machine learning to some differential equations. Specifically here, we are interested in non-canonical Hamiltonian problems, i.e. vector fields characterized by a symplectic form (non-canonical) and an invariant energy (Hamiltonian). Such problems include many plasma particle models and planar point vortices.

By learning a symplectic form and a Hamiltonian rather than a vector field without structure, the authors in [1] demonstrate that not only is short-time accuracy correct, but the long-time properties such as energy preservation are then also verified. However, they validate the learnt dynamics with simulations which use standard numerical methods that are ill-adapted for long-time simulation of such geometric problems. When using a more relevant scheme from [2], we surprisingly observed that the learnt problem behaves poorly for large time-steps, indicating that their method needs amending for long-time integration.

In this talk, we will present another method of learning, which is based on snapshots and numerical integrators. Crucially, the learnt dynamics are then compatible with geometric numerical schemes and therefore useful for long-time integration. This method will be validated on problems from plasma physics, notably the guiding center model. We will also detail some counter-intuitive properties of this method, offering insight on snapshot-based learning.

[1] Y. Chen, T. Matsubara, T. Yaguchi. *Neural Symplectic Form : Learning Hamiltonian Equations on General Coordinate Systems.* In *Advances in Neural Information Processing Systems*, vol. 34, pp. 16659–16670. Curran Associates, Inc., 2021.

[2] C. L. Ellison, J. M. Finn, J. W. Burby, M. Kraus, H. Qin, W. M. Tang. *Degenerate variational integrators for magnetic field line flow and guiding center trajectories.* Physics of Plasmas, **25(5)**, 052502, 2018. doi :10.1063/1.5022277.

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