Point-wise dissipation conditions in input-to-state stability of time-delay systems and Fredholm backstepping methods for spectral operators

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Motivation : The stability analysis in control theory, allows to know the behaviour of systems at infinity. For finite dimension systems, there exist some useful approaches to characterize stability which are very handy in practice. We are interesting of wheither we can extend some of these to certain infinite dimension systems.

I- Input-to-state stability of time-delay system. Consider the following class of time-delay system :

$$\dot{x}(t) = f(x_t, u(t)), \tag{1}$$

where $x_t \in C^0([-\Delta, 0], \mathbb{R}^n)$ is defined by $x_t(s) := x(t+s)$, $\Delta > 0$ is the maximum delay, u is the input and f is Lipschitz on bounded sets with $f(0,0) = 0$. In order to further the robustness properties of (1), we address its input-to-state stability (ISS), which is the stability that takes account of disturbances. The existing Lyapunov-Krasovskii functional (LKF) tool to ensure ISS for system (1) requires the so called LKF-wise dissipation meaning that the LKF V satisfies $\dot{V} \leq -\alpha(V) + \gamma(|u|)$. But, for input free system $(u = 0)$, only the point-wise dissipation ($\dot{V} \leq -\alpha(|x(t)|)$) is required to ensure global asymptotic stability for the system. As the dissipation rate α involves the current value of the system's solution in the point-wise dissipation, it is more easier to obtain in practice than the LKF one for which α involves the functional V itself. In [2], we provide some conditions to ensure ISS of (1) with the point-wise dissipation. Roughly speaking, we state the following :

Theorem 1. *Assume that there exists a LKF V which* ***dissipates point-wisely****. Then if the* ***dissipaton rate dominates its maximal increase along the system's solution at infinity****, the system* (1) *is ISS.*

We show through examples that the obtained conditions are more general than the existing techniques. Some other LKF-wise dissipation constructions will be mentionned [1].

II-Backstepping stabilization of PDE. Consider now the following control system

$$\partial_t v = \mathscr{A} v + Bw, \tag{2}$$

where $\mathscr{A}$ is a differential operator which generates a C^0 semigroup $e^{t\mathscr{A}}$, B is a given operator and w is the control. We would like to find a feedback law $w(t) = Kv(t)$ such that the system (2) is exponentially stable meaning all the solutions converge exponentially quickly to 0. A classical way to do that is by the so-called backstepping method which has met significant success for finite diemension systems. Using duality and compacteness analysis, this method was extended to some infinite dimension systems namely when $\mathscr{A}$ is skew-adjoint, self-adjoint, etc... But until now, there is no backstepping-based result for general spectral operator $\mathscr{A}$. In [3], we provide some conditions to stabilize (2) for spectral operator $\mathscr{A}$ in general using backstepping.

Theorem 2. *Consider the control system* (2). *Assume that the family of eigenvectors of $\mathscr{A}$ forms a Riesz basis and its eigenvalues λ_n satisfy $\lambda_n \sim n^\alpha$, $\alpha > 1$ and for any $\lambda > 0$, $|\lambda_n - \lambda_m + \lambda| \gtrsim |\lambda_n - \lambda_p| \gtrsim n^{\alpha-1}|n-m|$. Then, for any uniformely bounded from above and below B, there exists bounded linear feedback K such that the closed loop system $\partial_t v = \mathscr{A} v + BKv$ is exponentially stable.*

[1] E. Loko, A. Chaillet, and I. Karafyllis. Building coercive Lyapunov–Krasovskii functionals based on Razumikhin and Halanay approaches. *International Journal of Robust and Nonlinear Control*, 2024.

[2] E. Loko, A. Chaillet, Y. Wang, I. Karafyllis, and P. Pepe. Novel point-wise dissipation conditions to ensure input-to-state stability of time-delay systems. In process. 2024.

[3] E. Loko and A. Hayat. Fredholm backstepping for general linear systems. In process. 2024.

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