Laboratoire de Mathématiques de Reims

Modélisation du système urinaire inférieur de l'enfant.

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Summary

1. Context and Introduction
2. Modeling
3. Models and results

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1 Context and Introduction

2 Modeling

3 Models and results

Context.

American Memorial Hospital Foundation : Child Hospital of Reims

- Phd Lisa Grandjean.
- Grant : Patronage "Amis de l'hôpital Américain" Commitee.
- Three-dimensional modeling and numerical simulations of the child's lower urinary tract.
- Dates : 2022–2025
- Development of numerical models of the bladder's functions.

Purpose of this project

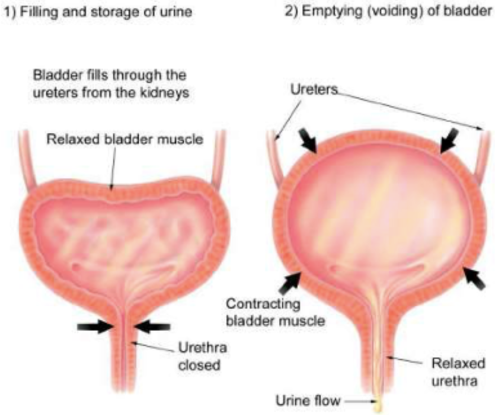
Provide physicians/medical doctors with auxiliary/complementary elements

- to "see what happens" in a non-invasive way,
- to understand normal behavior by highlighting principal features and parameters,
- then to infer development of diseases,
- to finally help diagnosis and treatment of diseases.

$\Longrightarrow$ "in silico" experiments !

- Medical multi-modal data available (Nadia Boudaoud's work) to build the model.
- Medical expertise

Lower urinary tract



Urine flow = water !

Detrusor = muscle, large deformations ! $\Longrightarrow$ **Fluid-structure interaction.**
State of the art : not a well-studied subject, not much for children.

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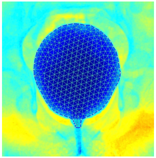
Medical data

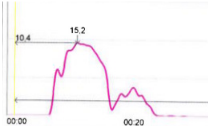
- Clinical data : size, volume from literature and study $\Longrightarrow$ 5 to 10 y.o bladders

Data required and sought	Data origin
Ureter diameter	Bibliography
Urethra diameter	Cystographic's measures/bibliography
Urethra Length	Cystographic's and per operative measures
Bladder volume	Koff formula : CV (mL) = (age+2) x 30
Urine inflow	Bibliography
Urine outflow	Flow measurement/Bibliography

- Cystography : dynamic X-ray (2D+t) of the bladder and urethra allowing to explore the walls of the bladder by injecting a contrast product - Mesh extracted from the cystography.



- UroDynamical results



Numerical method

Objective start from 2D models to move towards a 3D model.

Geometry extracted from images $\Longrightarrow$ unstructured meshes,
Finite element methods
$\Longrightarrow$ efficient and reliable method !

Free access, reproducible research, in-house code : based on FreeFem++ (http://www.freefem.org).

Navier-Stokes Equations in a moving domain

Navier-Stokes Equations (NS) with Arbitrary Lagrangian Eulerian (ALE) formulation

$$\begin{cases} \rho \left(\frac{\partial \vec{u}}{\partial t} + \underbrace{(\vec{u} - \vec{c}).\nabla \vec{u}}_{\text{inertia}} \right) - \underbrace{\mu \Delta \vec{u}}_{\text{viscous}} + \nabla p = 0 \quad \text{dans } \Omega^t \\ \nabla.\vec{u} = 0 \quad \text{dans } \Omega^t \end{cases} \tag{1}$$

ρ : density $(kg.m^{-3})$
μ : viscosity $(Pa.s)$
$\vec{u}$: velocity $(m.s^{-1})$
p : pressure $(Pa = kg.m^{-1}.s^{-2})$
$\vec{c}$: mesh velocity.

ALE Method

- Allows to solve the PDE in a **moving domain**.
- Build a map $\mathcal{A}^t$ from $\hat{\Omega}$ (reference domain) to Ω^t (current domain).

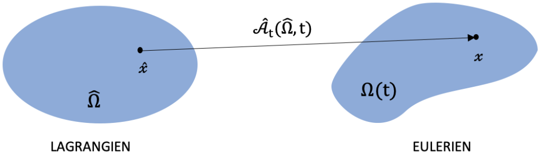


Figure: ALE map

$$\mathcal{A}_t : \overline{\hat{\Omega}} \to \mathrm{R}^2$$

where $\mathcal{A}_t$ continuous and bijective, $\hat{c}$ mesh velocity computed *e.g.* by harmonic extension.

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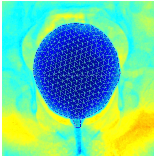
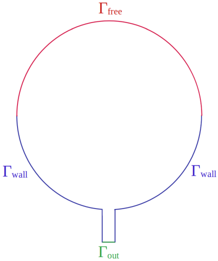
First Model

Emptying with free surface (Γ_t^{free}) : Spherical form (in 2D) when filled, vesical dome mobile, fixed part called trigone.



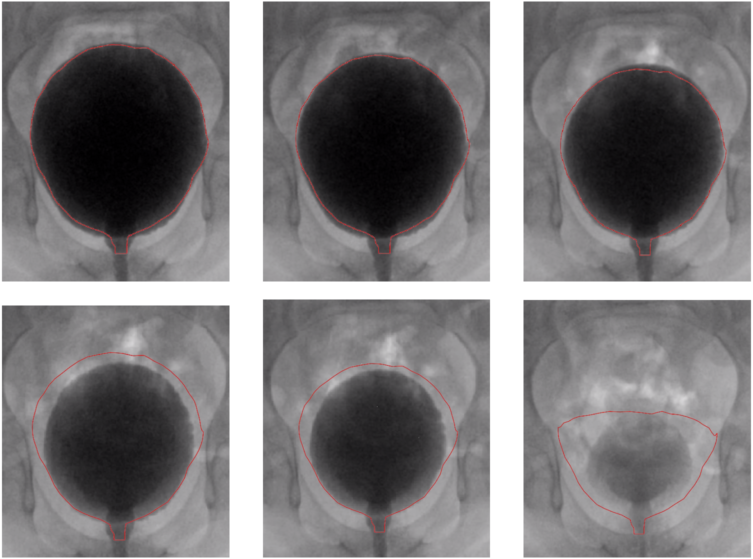


ALE Formulation (1) + boundary conditions :

$$\begin{cases} \vec{u} = 0 \quad \text{sur } \Gamma^{wall} \\ \vec{u} = \vec{u}_{out} \quad \text{sur } \Gamma^{out} \\ \nu \dfrac{\partial \vec{u}}{\partial \vec{n}} - p\vec{n} = -(p_e + \sigma(\kappa))\vec{n} \text{ sur } \Gamma_t^{free} \end{cases}$$

$\vec{u}_{out}$ such that emptying occurs in less than 30 s.
p_e : abdominal pressure ($\approx 13cmH2O$)
σ : air/water interface

Results



Discussions/limits

But :

- First model in **moving domain** with NS+ALE
- Filling by urethra is possible and realistic.
- Allows to reproduce cystography : filling and voidind by urethra.

Limits :

- Filling by ureters non realistic.
- Passive structure whereas for voiding active structure (contraction of the detrusor).
- Not so important fixed part.

2nd model : fluid with imposed displacement

- Fixed part
- Vesical dome : contact when the bladder is empty.

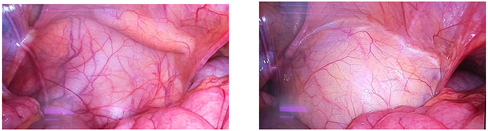


Figure: Empty bladder (left) and filled (right)

⇒ 2nd model : imposed displacement for reproducing movement observed in cystography.

2nd model : fluid with imposed displacement

Algorithm for filling :

While $V < Vmax$:

1. Compute displacement of the mesh $\vec{d}^{n+1}$
2. Solve fluid equations on Ω_n
3. Define the deformation
   $$T_n(\hat{x}) = \hat{x} + (\vec{d}^{n+1} - \vec{d}^n)$$
4. Compute new domain
   $$\Omega_{n+1} = T_n(\Omega_n)$$

3rd model : Fluid-Structure Interaction

- Fluid-Structure Interaction (FSI) : to find similar results as those with an imposed displacement.
- Numerically :

Monolithic : Treats fluid and structure dynamics in the same mathematical framework.
• High computational cost (possible in 2D but very expensive in 3D).
• Ensure well-posedness and convergence of the numerical model.

Partitioned : Treats fluid and structure as two computational fields.
• Reduce code development time and computational cost.
• Can be unstable, in particular when structure's density is close to fluid's density.

$\Longrightarrow$ **Our choice** : Monolithic approach.

Study of the structure

Structure :
$$\int_\Omega \rho \frac{\partial^2 U}{\partial t^2}.W dX + a(U,W) = \int_\Omega f.W dX \qquad (2)$$

Linear :

$$a(U,W) = \int_\Omega \sigma(U) : \nabla W dX$$

with

$$\sigma(U) = \lambda tr(U) Id + 2\mu\epsilon(U)$$
$$\epsilon(U) = \frac{1}{2}((\nabla U)^T + \nabla U)$$

Non linear :

$$a(U,W) = \int_\Omega F(U)\Sigma(U) : \nabla W dX$$

with

$$\mathrm{F(U)} = \mathrm{Id} + \nabla U$$
$$\Sigma(U) = \lambda tr(E(U)) Id + 2\mu E(U)$$
$$\mathrm{E(U)} = \frac{1}{2}((\nabla U)^T + \nabla U + (\nabla U)^T \nabla U)$$

$$\lambda = \frac{\nu E}{(1-2\nu)(1+\nu)}, \quad \mu = \frac{E}{2(1+\nu)}$$

Lamé coefficients from Young modulus de Young $E > 0$ and Poisson ratio $\nu \in]0, \frac{1}{2}[$

3rd model : Fluid-Structure Interaction

FSI Equations :

- NS + ALE (1) for fluid, in Ω_t^F
- Elasticity problem (**??**) (linear or not) for the structure, in Ω_t^S
- Coupling conditions on the interface Σ_t:

$$\begin{cases} \vec{u}^F = \vec{u}^S \quad \text{sur } \Sigma_t \\ \sigma^F(\vec{u}^F, p)\vec{n}^F = -\sigma^S(\vec{d}^S)\vec{n}^S \quad \text{sur } \Sigma_t \end{cases} \tag{3}$$

3rd model : Fluid-Structure Interaction

Monolithic formulation [1]

$$\Omega_n = \Omega_n^F \cup \Omega_n^S$$

$$u^{n+1} = \begin{cases} u^{F,n+1} \text{ in } \Omega_n^F \\ u^{S,n+1} \text{ in } \Omega_n^S \end{cases}$$

$$\begin{cases} \int_{\Omega_n^F} \rho^F \frac{u^{n+1}}{\Delta t}.v dx + \int_{\Omega_n^F} \rho^F (((u^n - c^n).\nabla) u^{n+1}).v dx - \int_{\Omega_n^F} (\nabla .v) p^{F,n+1} dx \\ + \int_{\Omega_n^F} 2\mu^F \epsilon(u^{n+1}) : \epsilon(v) dx + \int_{\Omega_n^F} (\nabla . u^{n+1}) q dx + \int_{\Omega_n^S} \rho^S \frac{u^{n+1}}{\Delta t}.v dx \\ + \int_{\Omega_n^S} L(u^{n+1}) : \nabla v dx = \int_{\Omega_n^{F,S}} \rho^{F,S} \frac{u^n}{\Delta t}.v dx + \int_{\Omega_n^{F,S}} \rho^{F,S} g.v dx \end{cases} \quad (4)$$

[1] Three-Dimensional Simulation of Fluid-Structure Interaction Problems Using Monolithic Semi-Implicit Algorithm. C.M Murea (2019)

Results - Non linear structure

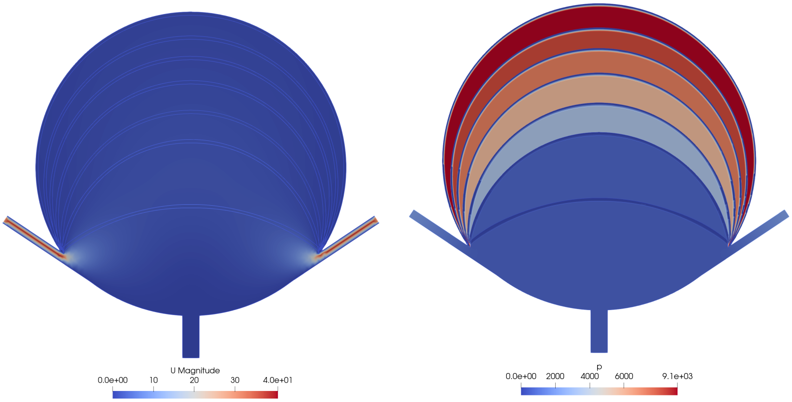


Figure: Velocity ($mm^3.s^{-1}$) Pressure ($miliPa$)

Discussions

- Uniform increasing pressure

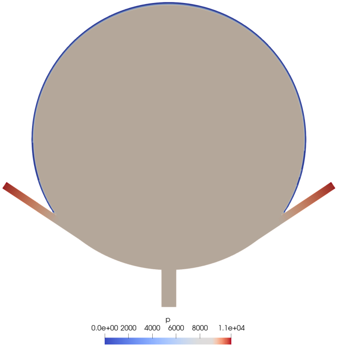
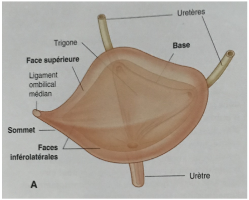


Figure: Pressure - Filled bladder

- What is the model for the structure ? What are the parameters ?

Conclusion and work in progress

- Different characteristic times for voiding ($\approx 30s$) and filling ($\approx 3h$).
- Different mechanisms : active structure for voiding/passive structure for filling.
- 3D simulations : complex geometry ...



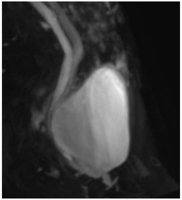


Figure: Scheme and 3D MRImage of a bladder.

Conclusion et perspectives

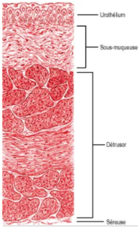


Figure: Structure study (Louise COTTON - 6th year Medecine study).

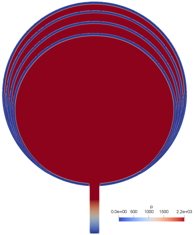


Figure: Active Structure.

Cassandre Logeart - 6th year Medecine study.
https://youtu.be/GhYNONlbwdA?feature=shar

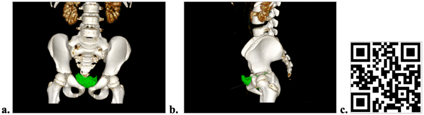


Figure 5 : Images de reconstruction de la vessie en trois dimensions à partir du logiciel