A robust preconditioner for saddle-point problems in an industrial context

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We consider the linear resolution of saddle-point systems arising from the discretisation of coupled or constrained systems. In many cases, such systems are challenging to solve by iterative methods and the developments of efficient preconditioners is an active field of research [2]. In this presentation, we present a robust block-preconditioner for a 2×2 block-system of the form

$$\begin{pmatrix} A & B^T \\ B & -C \end{pmatrix}, \tag{1}$$

with A symmetric positive definite (SPD) block, C positive semi-definite. Such problems have been extensively studied especially in academic communities and different families of preconditioners have been proposed [3].
In this talk, we highlight a problematic that has not been much investigated to the extent of our knowledge. In an industrial context, theoretical hypotheses are often not satisfied. Even the most simple systems of the form of Equation (1) can become challenging to solve for state-of-the-art linear solvers when the diagonal dominance of the block A is lost. Such a difficulty occurs for instance on distorted meshes. Industrial solvers then resort to direct solvers with all the ensuing limitations. To tackle this difficulty, we present an algebraic preconditioner with increased robustness for systems such as Equation (1). The key idea involves an algebraic transformation of the system that compensates the loss of diagonal dominance.
Since the approach we propose is algebraic in nature, it can be applied to a broad class of problems. In the context of the talk, we focus on systems encountered during the resolution of the incompressible Navier-Stokes equations discretised on general meshes with PolyMAC [1]. We first highlight the loss of robustness of classical approaches when considering distorted meshes. In a second time, we describe an innovative preconditioner based on an algebraic transformation of the linear system. Numerical results show impressive convergence on problems of industrial complexity.

Références

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[3] M. Benzi, A. J. Wathen. *Some Preconditioning Techniques for Saddle Point Problems*, pp. 195–211. Springer Berlin Heidelberg, Berlin, Heidelberg, 2008. doi :10.1007/978-3-540-78841-6_10.

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