Well-posedness of the Euler equations for a stratified ocean in isopycnal coordinates

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The dynamics of the ocean can be described by the incompressible Euler equations, completed with a transport equation for the density. For large scale dynamics, the ocean is assumed stratified, meaning that it is layered by sheets of constant density, namely isopycnals. This work is devoted to the study of perturbations around a shear flow equilibrium.
A fruitful viewpoint is the one of isopycnal coordinates, which uses the density as the vertical coordinate, flattening the isopycnals. The system becomes quasi-2D and thus is sometimes used in numerical studies [3].
While the well-posedness in Sobolev spaces has been treated in Eulerian coordinates (see [1]), a loss of derivatives appears in the reformulation in isopycnal coordinates, treated in [2] by the addition of a diffusion term. This work focuses on how to keep track of the symmetry of the system in Eulerian coordinates, while using the advantageous quasi-2D structure of the reformulation in isopycnal coordinates.
More precisely, the main result is a well-posedness theorem in Sobolev spaces on the system in isopycnal coordinates without diffusion. The time of existence is uniform with respect to the size of the perturbation, and with the additional assumptions of medium amplitude regime and small shear velocity, it is also uniform in the shallow-water parameter. This work is part of my PhD thesis under the supervision of Vincent Duchêne and David Lannes.

[1] B. Desjardins, D. Lannes, J.-C. Saut. *Normal mode decomposition and dispersive and nonlinear mixing in stratified fluids*, 2019. doi :10.48550/ARXIV.1912.05346.

[2] V. Duchêne, R. Bianchini. *On the hydrostatic limit of stably stratified fluids with isopycnal diffusivity*, 2022. doi :10.48550/ARXIV.2206.01058.

[3] S. M. Griffies, C. Böning, F. O. Bryan, E. P. Chassignet, R. Gerdes, H. Hasumi, A. Hirst, A.-M. Treguier, D. Webb. *Developments in ocean climate modelling.* Ocean Modelling, **2(3–4)**, 123–192, 2000. doi :10.1016/s1463-5003(00)00014-7.

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