# Auto-ajustement de la précision grâce au logiciel PROMISE

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# Introduction

Floating-point arithmetic: | Sign | Exponent | Mantissa |

Various floating-point formats:

| | #bits Mantissa ($p$) | Exp. | Range | $u = 2^{-p}$ |
|---|---|---|---|---|
| bfloat16 (half) | 8 | 8 | $10^{\pm 38}$ | $\approx 4 \times 10^{-3}$ |
| fp16 (half) | 11 | 5 | $10^{\pm 5}$ | $\approx 5 \times 10^{-4}$ |
| fp32 (single) | 24 | 8 | $10^{\pm 38}$ | $\approx 6 \times 10^{-8}$ |
| fp64 (double) | 53 | 11 | $10^{\pm 308}$ | $\approx 1 \times 10^{-16}$ |
| fp128 (quad) | 113 | 15 | $10^{\pm 4932}$ | $\approx 1 \times 10^{-34}$ |

↘ precision:

- ↘ execution time ☺
- ↘ volume of results exchanged ☺
- ↗ energy efficiency ☺

energy consumption proportional to $p^2$

| energy ratio | |
|---|---|
| fp64/fp32 | $\approx$ 5 |
| fp32/fp16 | $\approx$ 5 |
| fp32/bfloat16 | $\approx$ 9 |

- But computed results may be invalid because of rounding errors ☹

# Outline

In this talk we aim at answering the following questions.

1. How to control the validity of (mixed precision) floating-point results?
2. How to determine automatically the suitable format for each variable/part of a code?

# Rounding error analysis

## Several approaches

- Interval arithmetic
  - guaranteed bounds for each computed result
  - the error may be overestimated
  - specific algorithms
  - ex: INTLAB [Rump'99]
- Static analysis
  - no execution, rigorous analysis, all possible input values taken into account
  - not suited to large programs
  - ex: FLUCTUAT [Goubault & al'06], FLDLib [Jacquemin & al'19]
- Probabilistic approach
  - estimates the number of correct digits of any computed result
  - requires no algorithm modification
  - can be used in HPC programs
  - ex: CADNA [Chesneaux'90], SAM [Graillat & al'11], VERIFICARLO [Denis & al'16], VERROU [Févotte & al'17]

Classic arithmetic

$A \oplus B \longrightarrow R$

$R =$3.14237654356891

DSA

Random rounding

$A_1 \oplus B_1$  $\longrightarrow R_1$

$A_2 \oplus B_2$  $\longrightarrow R_2$

$A_3 \oplus B_3$  $\longrightarrow R_3$

$R_1 =$**3.14**1354786390989
$R_2 =$**3.14**3689456834534
$R_3 =$**3.14**2579087356598

- each operation executed 3 times with a random rounding mode

# Discrete Stochastic Arithmetic (DSA) [Vignes'04]

Classic arithmetic

$A \oplus B \longrightarrow R$

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- each operation executed 3 times with a random rounding mode
- number of correct digits in the results estimated using Student's test with the confidence level 95%

# Discrete Stochastic Arithmetic (DSA) `[Vignes'04]`

**Classic arithmetic**

$$A \oplus B \longrightarrow R$$

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**DSA**

Random rounding

$$A_1 \oplus B_1 \longrightarrow R_1$$
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- each operation executed 3 times with a random rounding mode
- number of correct digits in the results estimated using Student's test with the confidence level 95%
- operations executed synchronously
  - ⇒ detection of numerical instabilities
    Ex: `if (A>B)` with `A-B` numerical noise
  - ⇒ optimization of stopping criteria

# The CADNA library

cadna.lip6.fr

- implements stochastic arithmetic for C/C++ or Fortran codes
- provides stochastic types (3 floating-point variables and an integer)
- all operators and mathematical functions overloaded
  ⇒ few modifications in user programs
- support for MPI, OpenMP, GPU codes
- in one CADNA execution: accuracy of any result, complete list of numerical instabilities

[Chesneaux'90], [Jézéquel & al'08], [Lamotte & al'10], [Eberhart & al'18],...

# Stochastic types

Various stochastic types that can be mixed together or with classic types:

`half_st` `float_st` `double_st` `float128_st`

## Half precision in CADNA

control of fp16 computation with

- emulated half precision thanks to the library developed by C. Rau (`http://half.sourceforge.net`)
- native half precision on e.g. NVIDIA GPUs or recent ARM processors (successful tests on Fugaku supercomputer)

# Numerical validation... and then?

Can we use reduced or mixed precision
to improve performance and energy efficiency?

- mixed precision linear algebra algorithms
  - matrix-matrix and matrix-vector multiplication
  - LU and QR matrix factorizations
  - iterative refinement
  - Krylov solvers
  - least squares problems

  survey: [Higham & Mary'22]

- precision autotuning

# Precision autotuning

## Static tools

- FPTaylor/FPTuner [Solovyev & al'15] symbolic Taylor expansions
- DAISY [Darulova & al'18] mixed-precision with rewriting
- TAFFO [Cherubin & al'19] auto-tuning for floating to fixed-point optimization
- POP [Ben Khalifa & al'19] error analysis by constraint generation

not suited to large scale programs ☹

# Precision autotuning

## Dynamic tools

intend to deal with large codes

- CRAFT [Lam & al'13] binary modifications on the operations
- Precimonious [Rubio-Gonzàlez & al'13] source modification with LLVM
- Blame Analysis [Nguyen & al'15] improves Precimonious
- HiFPTuner [Guo & al'18] based on a hierarchical search algorithm
- ADAPT [Menon & al'18] based on algorithmic differentiation
- FloatSmith [Lam & al'19] combination of CRAFT & ADAPT
- Tools dedicated to GPUs (that pay attention to casts):
  - AMPT-GA [Kotipalli & al'19]
  - GPUMixer [Laguna & al'19]
  - GRAM [Ho & al'21]

# Precision autotuning

Dynamic tools rely on comparisons with the highest precision result.

⚠ [Rump'88] $P = 333.75y^6 + x^2(11x^2y^2 - y^6 - 121y^4 - 2) + 5.5y^8 + x/(2y)$
with $x = 77617$ and $y = 33096$

float: $P =$2.571784e+29

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float: $P =$2.571784e+29
double: $P =$1.17260394005318
quad: $P =$1.17260394005317863185883490452018

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float: $P =$2.571784e+29
double: $P =$1.17260394005318
quad: $P =$1.17260394005317863185883490452018
exact: $P \approx$-0.827396059946821368141165095479816292

# PROMISE (PRecision OptiMISE) [Graillat & al'19], [F.J. & al'21]

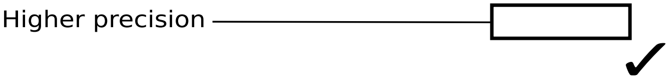
promise.lip6.fr

PROMISE

- provides a mixed precision code (half, single, double) taking into account a required accuracy
- uses CADNA to validate a type configuration
- uses the Delta Debug algorithm `[Zeller'09]` to search for a valid type configuration with a mean complexity of $O(n \log(n))$ for $n$ variables.

# Searching for a valid configuration with 2 types

Method based on the Delta Debug algorithm `[Zeller'09]`



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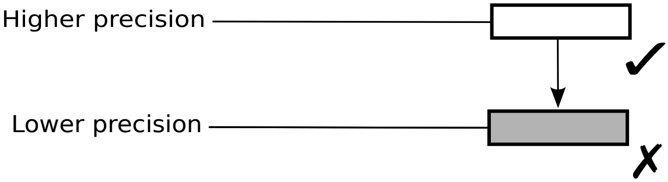

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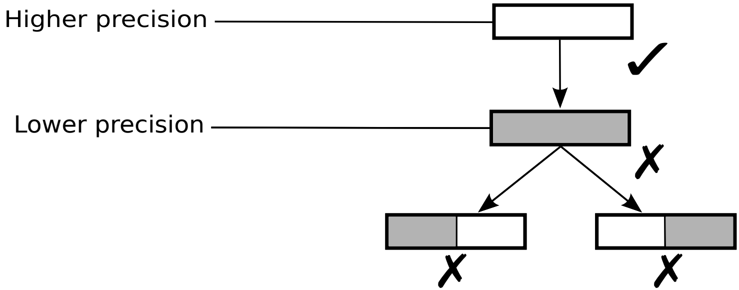

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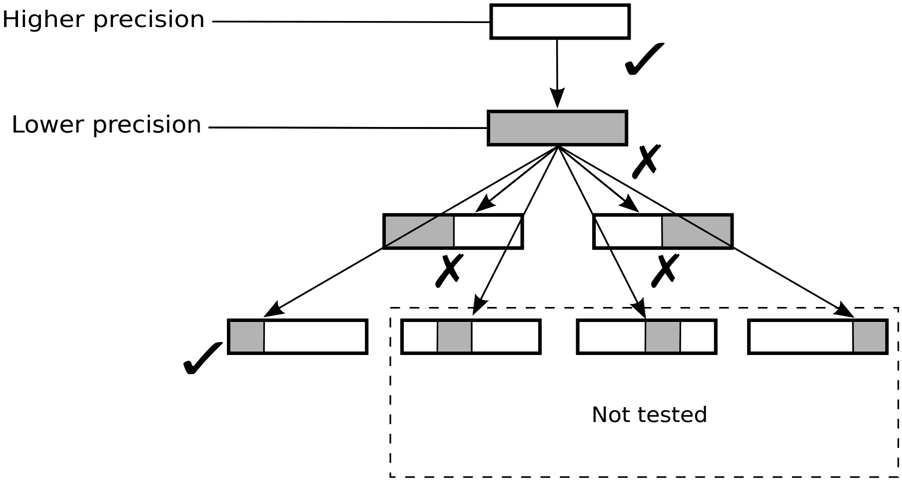

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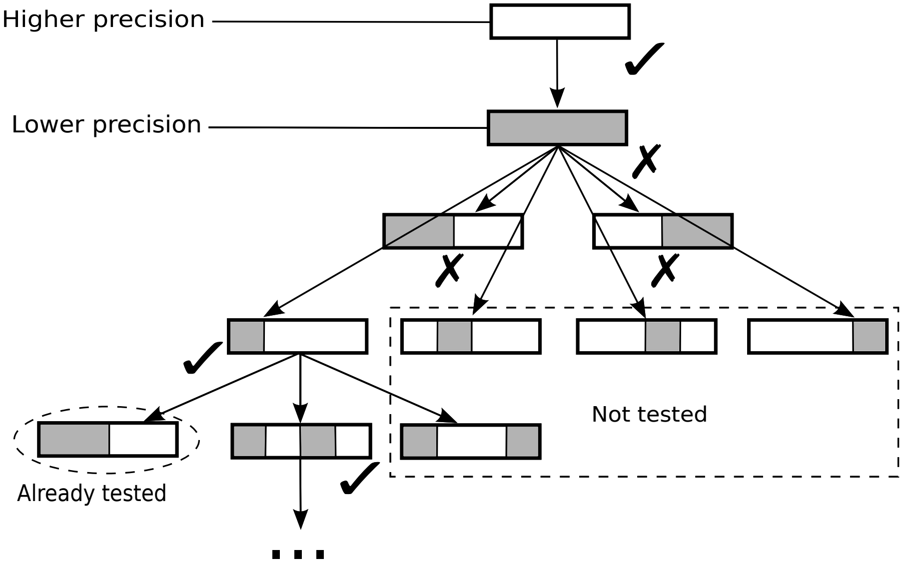
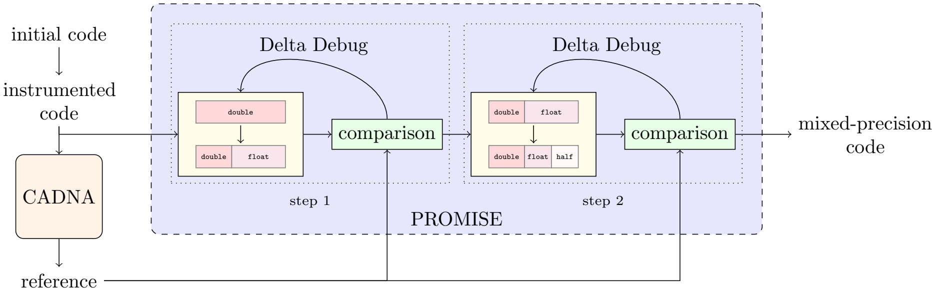


# PROMISE in double, single and half precision




- step 1: code in double → variables relaxed to single precision
- step 2: single precision variables → variables relaxed to half precision

# MICADO: simulation of nuclear cores

code developed by EDF (French energy supplier)

- neutron transport iterative solver
- 11,000 C++ code lines

| # req. digits | # single - # double | speed up | memory gain |
|---|---|---|---|
| 10 | 32-19 | 1.01 | 1.00 |
| 8 | 33-18 | 1.01 | 1.01 |
| 6 | 38-13 | 1.20 | 1.44 |
| 5<br>4 | 51-0 | 1.32 | 1.62 |

- Speedup, memory gain w.r.t. the double precision version
- Speed-up up to 1.32 and memory gain 1.62
- Mixed precision approach successful: speed-up 1.20 and memory gain 1.44

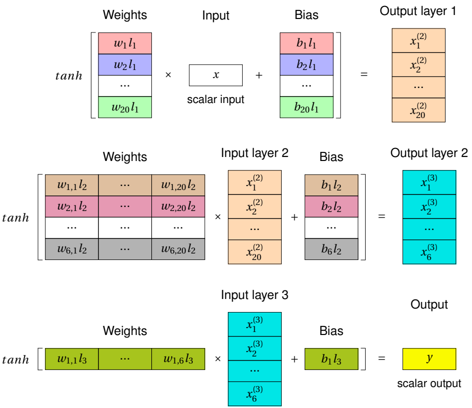
# Precision autotuning of neural networks

- neural networks created and trained with Keras or PyTorch
- automatically transformed into C++ codes to be used with PROMISE

Sine NN: interpolation network approximating the sine function

- Scalar input/ouput
- 3 dense layers with tanh activation function:
  - 20 neurons → 21 types to set
  - 6 neurons → 7 types to set
  - 1 neuron → 2 types to set

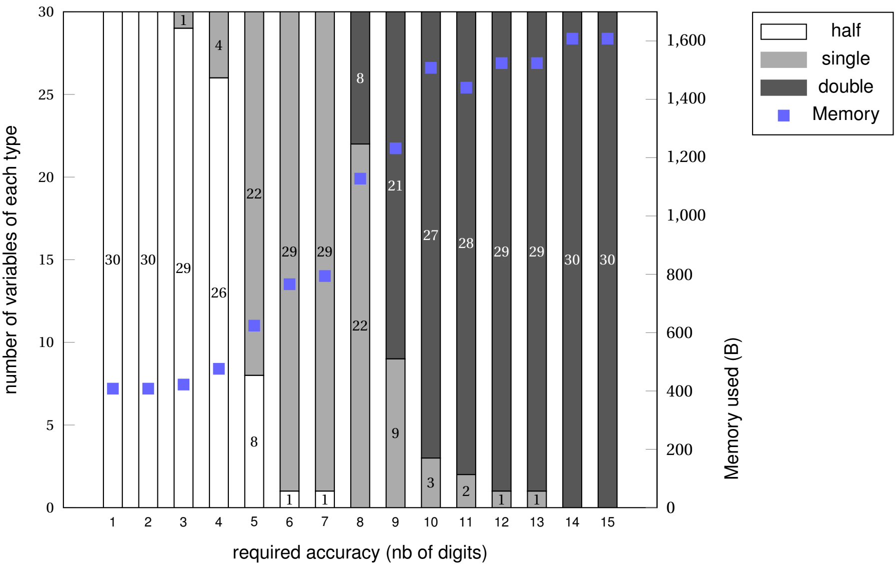
⇒ 30 types to set in total



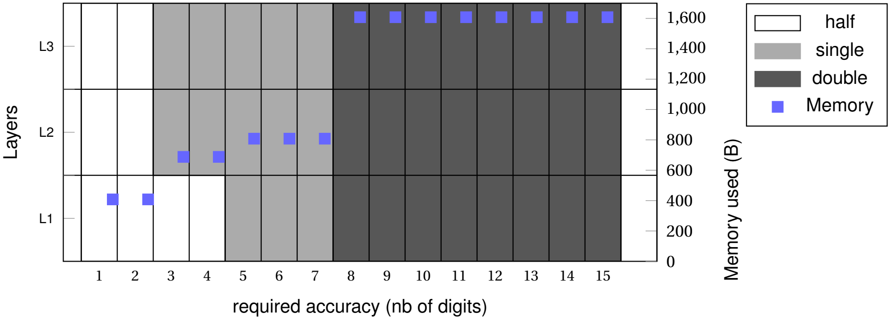

2 approaches:

- one type per neuron
- one type per layer

# Sine NN, one type per neuron



# Sine NN, one type per layer




In this talk, input=0.5
similar trends observed with different input values

# MNIST NN

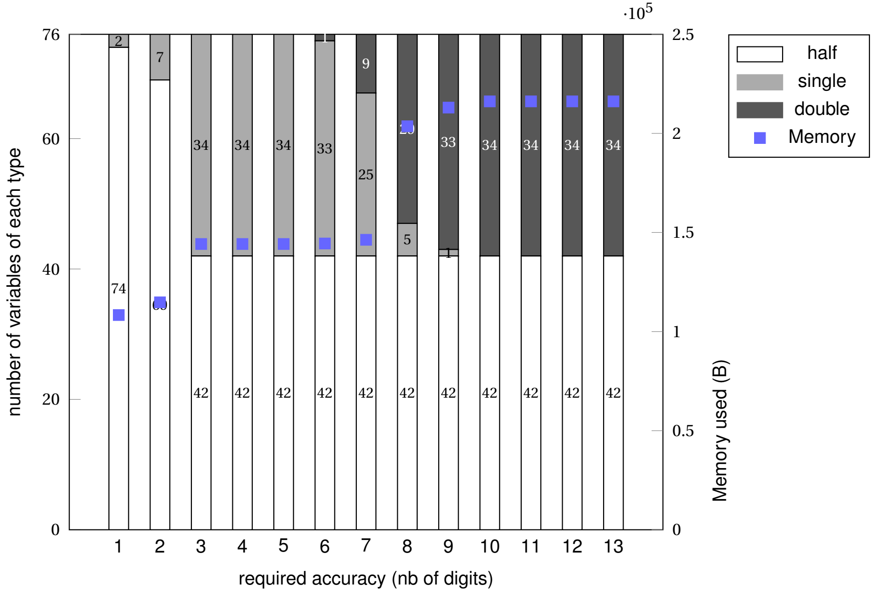
Classification of handwritten digits:

- input: vector of size 784 (flatten image)
- output : vector of size 10, probability distribution for the 10 different classes
- 2 dense layers:
  - 64 neurons and ReLU activation function → 65 types to set
  - 10 neurons and softmax activation function → 11 types to set

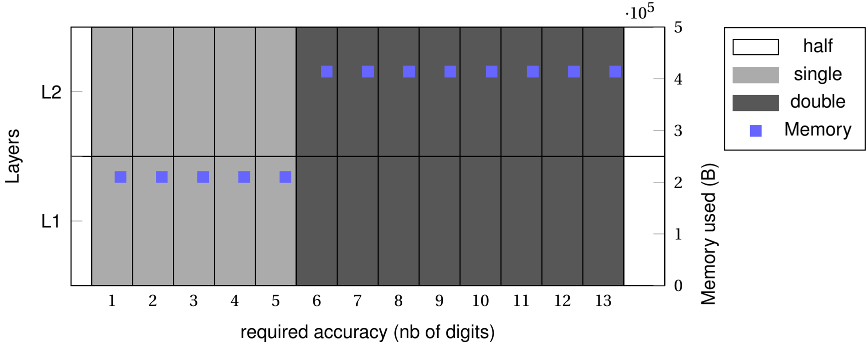
wikipedia.org

⇒ 76 types to set in total

# MNIST NN, one type per neuron



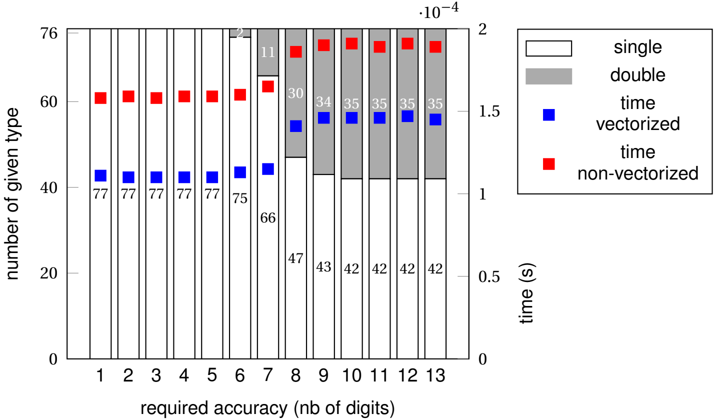
# MNIST NN, one type per layer




In this talk, input image test_data[61] from MNIST data base
similar trends observed with different input images

# Time gain with MNIST NN, one type per neuron

matrix-vector products vectorized using OpenMP SIMD on AVX2
⇒ 8 fp32 (or 4 fp64) operations in parallel

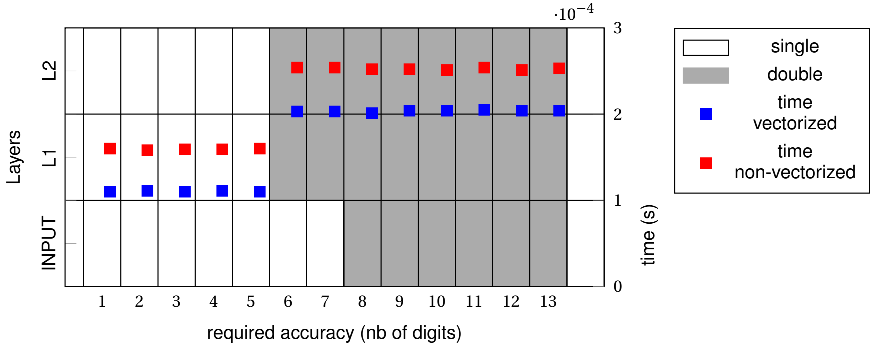



time ratio fp64 / mixed precision:

- up to 1.30 for non-vectorized codes
- up to 1.41 for vectorized codes

theoretical time ratio fp64 / fp32 = 2

# Time gain with MNIST NN, one type per layer




time ratio fp64 / fp32:

- up to 1.60 for non-vectorized codes
- up to 1.86 for vectorized codes

time ratio non-vectorized / vectorized: up to 1.45

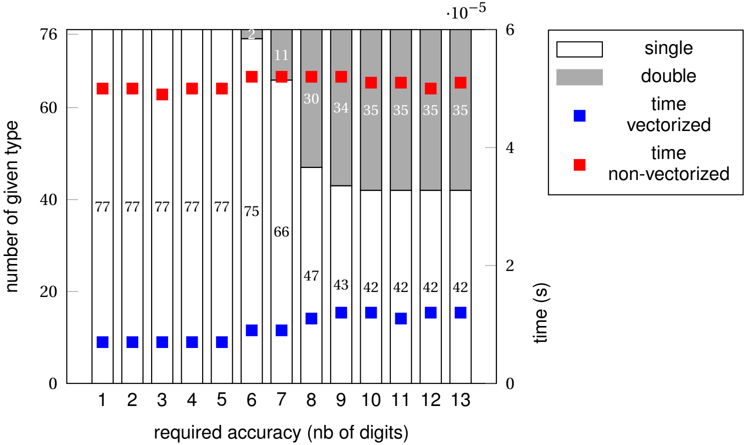
theoretical time ratio:

fp64 / fp32 = 2

non-vect. / vect. = 8 in fp32, 4 in fp64

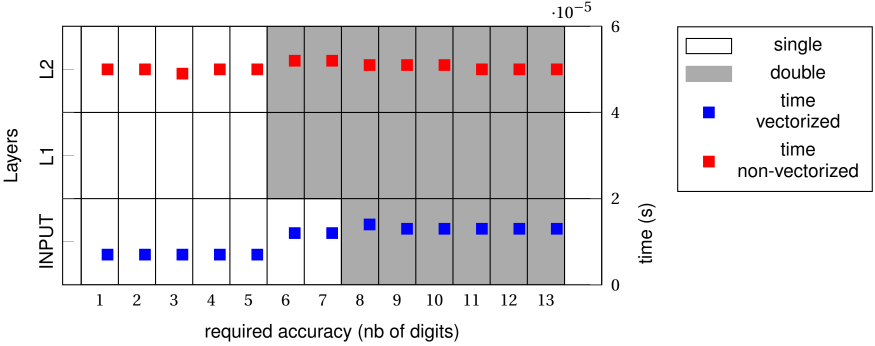
# Matrix-vector products in MNIST NN

## one type per neuron



# Matrix-vector products in MNIST NN

one type per layer

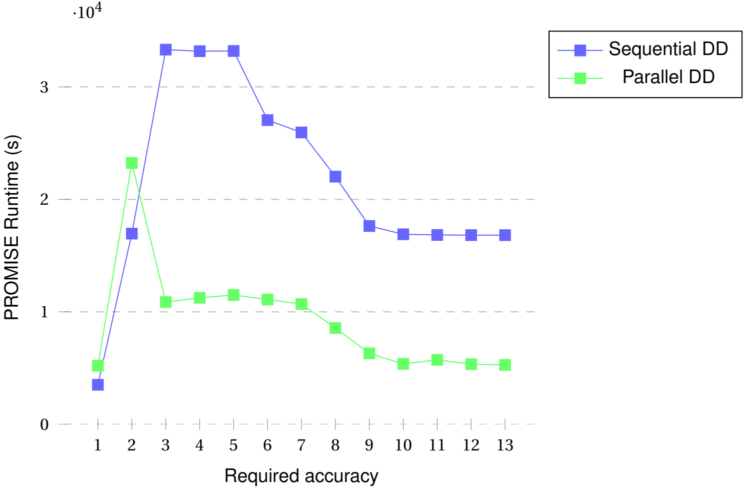



time ratio fp64 / fp32:

- no time difference for non-vectorized codes
- 2 for vectorized codes

time ratio non-vectorized / vectorized: up to 7.2 in fp32 and 3.9 in fp64
☺ close to theoretical ratio

# Execution time of PROMISE for MNIST NN




Parallel version of the Delta Debug from [Hodován & Kiss'16]

Speedup up to 3.2 on a CPU having 6 cores with 16 GB RAM

# Conclusion/Perspectives

## Conclusion

in user codes, suitable configuration types provided by precision autotuning

- $\searrow$ memory consumption
- $\searrow$ execution time, in particular in vectorized codes

## Perspectives

- extension of CADNA/PROMISE to other formats such as bf16
- extension of PROMISE to GPUs
- floating-point autotuning in arbitrary precision
- combination of mixed precision algorithms and floating-point autotuning

PostDoc offer in LIP6!

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Thank you for your attention!
Any question?