Numerical solution of elliptic partial differential equation problems in high dimension using two-layer neural networks

Virginie EHRLACHER, CERMICS - Champs-sur-Marne
Mathias DUS, CERMICS - Champs-sur-Marne

The aim of this work is to analyze numerical schemes using two-layer neural networks with infinite width for the resolution of the high-dimensional Poisson partial differential equation (PDE) with Neumann boundary condition. Using Barron's representation of the solution with a probability measure defined on the set of parameter values, the energy is minimized thanks to a gradient curve dynamic on the 2-Wasserstein space of the set of parameter values defining the neural network. Inspired by the work from Bach and Chizat [1], we prove that if the gradient curve converges, then the represented function is the solution of the elliptic equation considered. In contrast to previous works, the activation function we use here is not assumed to be homogeneous to obtain global convergence of the flow. Numerical experiments are given to show the potential of the method.
Link to the preprint : ml_edp
In addition, I will present some results about the efficiency of neural networks to solve elliptic eigenvalue problems in high dimension.

[1] F. Bach, L. Chizat. *On the global convergence of gradient descent for over-parameterized models using optimal transport.* In *Advances in Neural Information Processing Systems*, vol. 31. Curran Associates, Inc., 2018.

Contact : mathias.dus@enpc.fr