Well-posedness of a non-local ocean-atmosphere coupling model

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Ocean-atmosphere interactions play a critical role in climate modeling and weather forecasting. Ocean and atmosphere models have been constructed separately by two distinct communities and numerical models couple them with a complex interface. We propose a translation of this coupled system into a global mathematical model in order to use the tools of analysis and study its well-posedness. We present a simplified model but realistic, i.e. containing the main ingredients of numerical models. This mathematical model is known as the coupled Ekman problem, considering vertical exchanges of the horizontal currents $\mathbf{u}_\alpha$ with $\alpha \in \{o, a\}$, the coriolis effect f, and the effect of small scales via turbulent viscosities ν. The particularity of this model is to consider the interface as a buffer zone delimited by the altitudes $\delta_o < 0 < \delta_a$ with interface conditions specific to the ocean-atmosphere coupling. These interface conditions lead to the dependence of viscosity profiles on the jumps of the current around the interface and make the global problem non-local :

$$
\begin{aligned}
\partial_t \mathbf{u}_\alpha - \partial_z \left(\nu_\alpha(z, u^*(t)) \partial_z \mathbf{u}_\alpha\right) &= \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} (\mathbf{u}_\alpha^g - \mathbf{u}_\alpha) && \text{sur } (\delta_\alpha, z_\alpha^\infty) \times]0, T[, \\
\mathbf{u}_\alpha(z_\alpha^\infty, t) &= \mathbf{u}_\alpha^g(z_\alpha^\infty, t) && \text{sur }]0, T[, \\
\mathbf{u}_\alpha(z, t = 0) &= \mathbf{u}_\alpha^0(z) && \text{sur } [z_o^\infty, z_a^\infty], \\
\nu_o \, \partial_z \mathbf{u}_o(\delta_o, t) &= \lambda^2 \nu_a \, \partial_z \mathbf{u}_a(\delta_a, t) && \text{sur }]0, T[, \\
\nu_a \, \partial_z \mathbf{u}_a(\delta_a, t) &= C_D \left\| \mathbf{u}(\delta_a, t) - \mathbf{u}(\delta_o, t) \right\| \left(\mathbf{u}(\delta_a, t) - \mathbf{u}(\delta_o, t)\right) && \text{sur }]0, T[, \\
u^*(t) &= \sqrt{C_D} \left\| \mathbf{u}_a(\delta_a, t) - \mathbf{u}_o(\delta_o, t) \right\| && \text{sur }]0, T[.
\end{aligned}
$$

To study the well-posedness of this system, a first method is rewrite it as a fixed point problem in order to deal with the non-local aspects. A general study of the problem in its stationary and unsteady state leads to a sufficient condition to guarantee the well-posedness. A simplified version of this condition can be expressed as :

$$
\left\| \frac{\partial \nu_\alpha}{\partial u^*} \right\|_{\mathcal{L}^\infty} \sqrt{\left\| \sqrt{\nu_a} \partial_z \mathbf{u}_a \right\|_2^2 + \lambda^{-2} \left\| \sqrt{\nu_o} \partial_z \mathbf{u}_o \right\|_2^2} \leq C \min_{z, u^*} \nu_\alpha(z, u^*)
$$

This condition applied to the ocean-atmosphere framework, i.e. with physically-realistic viscosity profiles and orders of magnitude, is too restrictive and does not guarantee the uniqueness of solutions. In the stationary case, a necessary and sufficient condition can be given to ensure the existence and uniqueness of solutions. We will see that, once again, in the context of ocean-atmosphere coupling, this condition is not met and there is no uniqueness of solutions. In conclusion, we will discuss the prospects for such a model and the parameters that could be adjusted to obtain a mathematically well-posed model. Details of the proofs of this study are presented in [1].

[1] S. Thery. *Well-posedness of a non local ocean-atmosphere coupling model : study of a 1d ekman boundary layer problem with non local kpp-type turbulent viscosity profile*. Submitted, 2023. <hal-04327972>.

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