Mathematical models for flagellar activation

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In order to compensate the lack of inertia in a fluid at a low Reynolds number, strategies involving non-reciprocal movement patterns need to be employed by swimmers at the microscopic scale [5]. Among them, the most common one is using beating filaments such as cilia, or flagella which has a characteristic pattern to maximize swimming efficiency. However, this motion cannot be easily reproduced when studying a simple elastic filament without any form of activation along is length. In this case, the wave propagating along the swimmer is attenuated very quickly [2, 4], which does not match the behaviors observed in the tails of swimming micro-organisms in biology, or in data-based simulations [7].

In practice, an entire structure propagating curvature variations is present along the flagellum. This structure, called axoneme, is mainly composed of pairs of filaments arranged in a circle, between which molecular motors walk [3, 6]. These motors are the active component which create bending along the flagellum, and are powered by a chemical component called Adenosine triphospate (ATP).
In this talk, I will present a detailed mathematical model of the axoneme. I will particularly focus on the molecular motors' behavior, first through a two-state model, governed by Fokker-Planck equations. I will then introduce other more complex models to take into account the whole structure of this axoneme. I will study mathematically and numerically the influence of the ATP quantity on the system's behavior, and discuss its biophysical interpretation [1].

This work was conducted with F. Alouges (ENS Paris-Saclay), I. Anello (SISSA), A. DeSimone (SISSA) and A. Lefebvre-Lepot (CentraleSupélec).

[1] F. Alouges, I. Anello, A. DeSimone, A. Lefebvre-Lepot, J. Levillain. *Inside an n-axoneme : a mathematical model for flagellar activation mechanisms.* to be submitted, 2024.

[2] F. Alouges, A. Lefebvre-Lepot, J. Levillain. *A limiting model for a low reynolds number swimmer with n passive elastic arms.* Mathematics in Engineering, **5(5)**, 1–20, 2023.

[3] F. Jülicher, J. Prost. *Cooperative molecular motors.* Phys. Rev. Lett., **75**, 2618–2621, 1995.

[4] K. E. Machin. *Wave Propagation along Flagella.* Journal of Experimental Biology, **35(4)**, 796–806, 1958.

[5] E. M. Purcell. *Life at low Reynolds number.* American Journal of Physics, **45(1)**, 3–11, 1977.

[6] P. Sartori, V. F. Geyer, J. Howard, F. Jülicher. *Curvature regulation of the ciliary beat through axonemal twist.* Phys. Rev. E, **94**, 042426, 2016. doi :10.1103/PhysRevE.94.042426.

[7] B. J. Walker, S. Phuyal, K. Ishimoto, C.-K. Tung, E. A. Gaffney. *Computer-assisted beat-pattern analysis and the flagellar waveforms of bovine spermatozoa.* R. Soc. Open Sci., **7(6)**, 2020.

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