Multi-scale analysis and reduced models for low-Reynolds swimmers

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The topic of this presentation is a multi-timescale approach to derive reduced models of particles and swimmers in a viscous (low Reynolds-number) fluid.
Over a long period of time, or from a distance, the trajectory of self-propelling bodies such as swimmers appears smooth, with their trajectories appearing almost ballistic. This long-time behaviour, however, masks more complex dynamics, such as the side-to-side snakelike motion exhibited by spermatozoa as they swim, propelled by the frequent and periodic beating of their flagellum, or shape-changing microorganisms and microrobots. Many models of motion at microscopic scale, such as the celebrated Jeffery equations established in 1922 [3], neglect, often without formal justification, these effects in favour of smoother long-term behaviours.
In this talk, I will present recent results based on multi-timescale analysis and evaluating the long-term effects of high-frequency oscillations on translational and angular motion for various classical swimming models of micro-scale swimmers, with the purpose of assessing the relevance of neglecting these oscillations, and derive simplified equivalent models.
I will particularly focus on Jeffery equations and subsequent generalisations [2]. The simplest formulation of Jeffery equation deals with the planar motion of an ellipsoidal particle in a shear flow, whose position (x, y) and orientation θ are governed by the following dynamical system :

$$\begin{cases} \dot{x} = V\cos\theta + \gamma y, \\ \dot{y} = V\sin\theta, \\ \dot{\theta} = \frac{\gamma}{2}(-1 + B\cos(2\theta)), \end{cases} \tag{1}$$

with V, B and γ respectively representing the particle's velocity, its elongation and the shear flow strength.
When adding a fast-timescale term in the orientational dynamics, represented by some $f(\omega t)$ with $\omega \gg 1$, one can show that the averaged system still follows Jeffery trajectories, with effective parameters explicitly calculated from the function f. I will detail the multiscale method on this example and discuss its physical interpretation, and describe extensions to deformable particles and three-dimensional motion [4, 1].
This work was conducted with M. Dalwadi (UCL), E. Gaffney (Oxford University), K. Ishimoto (Kyoto University), and B. Walker (University of Bath).

[1] E. A. Gaffney, M. P. Dalwadi, C. Moreau, K. Ishimoto, B. J. Walker. *Canonical orbits for rapidly deforming planar microswimmers in shear flow.* Physical Review Fluids, **7(2)**, L022101, 2022.

[2] K. Ishimoto. *Jeffery's orbits and microswimmers in flows : A theoretical review.* Journal of the Physical Society of Japan, **92(6)**, 062001, 2023.

[3] G. B. Jeffery. *The motion of ellipsoidal particles immersed in a viscous fluid.* Proceedings of the Royal Society of London. Series A, Containing papers of a mathematical and physical character, **102(715)**, 161–179, 1922.

[4] B. J. Walker, K. Ishimoto, E. A. Gaffney, C. Moreau, M. P. Dalwadi. *Effects of rapid yawing on simple swimmer models and planar jeffery's orbits.* Physical Review Fluids, **7(2)**, 023101, 2022.

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