Optimal computational boundary conditions for linear elliptic equations in random media

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We are interested in computing the electric field ∇u generated by a neutral charge distribution $\nabla \cdot h$ in a random environment, that is modelled by the following divergence form elliptic equation

$$-\nabla \cdot a\nabla u = \nabla \cdot h \quad \text{in } \mathbb{R}^3 \quad \text{with } u(x) \underset{|x|\uparrow\infty}{\to} 0, \tag{1}$$

where the conductivity a is sampled from a stationary and ergodic probability measure $\mathbb{P}$. For numerical purposes we compute the solution of (1) in a box $[-L,L)^d$ for $L \gg 1$ with a Dirichlet boundary condition, that is

$$\begin{cases} -\nabla \cdot a\nabla u_L = \nabla \cdot h & \text{in } [-L,L)^d, \\ u_L = u_{\mathrm{bc}} & \text{on } \partial[-L,L)^d, \end{cases}$$

where the goal is to find the boundary condition u_{bc} that leads to the optimal convergence rate for $\|\nabla u - \nabla u_L\|_{\mathrm{L}^2([-L,L)^d)}$ as $L \uparrow \infty$. I will explain how to build the optimal boundary condition, that combines ideas from the theory of effective multipoles in random environment developed by Bella, Giunti and Otto [1] as well as the homogenization theory for linear ellitpic systems and recent estimates for the semi-group established in [2]. This is based on the preprint [3].

[1] P. Bella, A. Giunti, F. Otto. *Effective multipoles in random media.* Communications in Partial Differential Equations, **45(6)**, 561–640, 2020.

[2] N. Clozeau. *Optimal decay of the parabolic semigroup in stochastic homogenization for correlated coefficient fields.* Stochastics and Partial Differential Equations : Analysis and Computations, **11(3)**, 1254–1378, 2022.

[3] N. Clozeau, L. Wang. *Artificial boundary conditions for random ellitpic systems with correlated coefficient field.* arXiv preprint arXiv :2309.06798, 2023.

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