# Dictionary-based model reduction for state estimation

**Anthony NOUY**, LMJL - Nantes **Alexandre PASCO**, LMJL - Nantes

We consider in [4] the problem of state estimation from $m$ linear measurements, where the state $u$ to recover is an element of the manifold $\mathcal{M}$ of solutions of a parameter-dependent equation. The state is estimated using prior knowledge on $\mathcal{M}$ coming from model order reduction. Variational approaches based on linear approximation, such as [3], yields a recovery error limited by the Kolmogorov $m$-width of $\mathcal{M}$. To overcome this issue, piecewise-affine approximations [2] of the manifold have also be considered, that consist in using a library of linear spaces among which one is selected by minimizing some distance to the manifold.
In [4] we propose a state estimation method relying on dictionary-based model reduction, where a space is selected from a library generated by a dictionary of snapshots, using a distance to the manifold. The selection is performed among a set of candidate spaces obtained from the path of a $\ell_1$-regularized least-squares problem. Then, in the framework of parameter-dependent operator equations (or PDEs) with affine parameterization, we provide an efficient offline-online decomposition based on randomized linear algebra [1], that ensures efficient and stable computations while preserving theoretical guarantees.

[1] O. Balabanov, A. Nouy. *Randomized linear algebra for model reduction—part II : Minimal residual methods and dictionary-based approximation.* Adv Comput Math, **47(2)**, 26, 2021. doi : 10.1007/s10444-020-09836-5.

[2] A. Cohen, W. Dahmen, O. Mula, J. Nichols. *Nonlinear Reduced Models for State and Parameter Estimation.* SIAM/ASA J. Uncertainty Quantification, **10(1)**, 227–267, 2022. doi : 10.1137/20M1380818.

[3] Y. Maday, A. T. Patera, J. D. Penn, M. Yano. *A parameterized-background data-weak approach to variational data assimilation : Formulation, analysis, and application to acoustics.* Int. J. Numer. Meth. Engng, **102(5)**, 933–965, 2015. doi :10.1002/nme.4747.

[4] A. Nouy, A. Pasco. *Dictionary-based model reduction for state estimation*, 2023. doi : 10.48550/ARXIV.2303.10771.

Contact : alexandre.pasco@ec-nantes.fr