Numerical insights onto the generalized Steklov problem

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In this talk, we present our numerical results on the generalized Steklov problem for the modified Helmholtz equation and focus on the relation between its spectrum and the geometric structure of the domain. We consider both the interior problem (when the domain is bounded), and the exterior problem (exterior of a bounded set) in two and three dimensions. For this study, we implemented an original finite-element method that solves the modified Helmholtz equation and constructs a matrix representation of the Dirichlet-to-Neumann operator to be diagonalized for approximating the eigenvalues and eigenfunctions of the operator [1]. For the exterior (unbounded) domain we also use a perfectly matched layer technique. We address three distinct aspects: (i) the asymptotic behavior of eigenvalues; (ii) the dependence of the integrals of eigenfunctions on the domain symmetries; and (iii) the localization and decay of Steklov eigenfunctions away from the boundary. Finally, we discuss applications of the obtained results in the theory of diffusion-controlled reactions and formulate conjectures with relevance in spectral geometry.

[1] A. Chaigneau, D. S. Grebenkov. *A numerical study of the Dirichlet-to-Neumann operator in planar domains.* (submitted) preprint arXiv:2310.19571, 2023.

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