Weighted and uniform optimal control of ensembles of systems via Γ-convergence

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In this presentation, we consider an ensemble of affine-control systems in $\mathbb{R}^n$ of the form

$$\dot{x} = A^\theta(x)u + b^\theta(x),$$

parametrised by $\theta \in \Theta$ (compact subset of an Euclidean space), which are simultaneously driven on the time horizon $[0, T]$ by the same control $u \in \mathcal{U} := L^2([0, T], \mathbb{R}^m)$. Given a probability measure $\mu \in \mathcal{P}(\Theta)$ that quantifies our uncertainty on the parameter θ, we consider the weighted functional $J_\mu : \mathcal{U} \to \mathbb{R}$ defined as

$$J_\mu(u) := \int_\Theta a(x_u^\theta(T), \theta) d\mu(\theta) + \lambda \|u\|_{L^2}^2,$$

where $\lambda > 0$ tunes the regularisation, and $a : \mathbb{R}^n \times \Theta \to \mathbb{R}$ represents the end-point cost for the elements of the ensemble. Considering a sequence $(\mu_n)_n$ weakly converging to μ (e.g., obtained through samplings/experiments), we show that the respective functionals $(J_{\mu_n})_n$ are Γ-converging to J_μ. This implies the strong convergence of the minimisers $\hat{u}_n$ of J_{μ_n} to the minimisers of J_μ.
An analogous result can be proved for the uniform optimal control problem

$$G(u) := \sup_{\theta \in \Theta} a(x_u^\theta(T), \theta) + \lambda \|u\|_{L^2}^2.$$

Indeed, if we have a sequence of triangulations $(\Theta_n)_n$ that properly approximate Θ, then the functionals

$$G_n(u) = \sup_{\theta \in \Theta_n} a(x_u^\theta(T), \theta) + \lambda \|u\|_{L^2}^2$$

are Γ-convergent to G. Finally, we will see how the first order necessary optimality conditions relate every uniform optimal control problem to a weighted one. The material presented can be found in [1, 2].

[1] A. Scagliotti. *Optimal control of ensembles of dynamical systems.* ESAIM : Control Optim. Calc. Var., **29(22)**, 2023.

[2] A. Scagliotti. *Minimax problems for ensembles of affine-control systems.* In preparation, 2024.

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