Extrapolation in the Wasserstein space

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Given two probability measures ν_0 and ν_1 on $\mathbb{R}^d$ with bounded second moments, there always exists a length-minimizing geodesic connecting the two with respect to the L^2 Wasserstein metric. This gives a natural notion of interpolation between the two measures. In general, however, it is not possible to extend such geodesics while keeping them length minimising, since particles may collide just after reaching the arrival measure. In this talk, we will consider a variational problem that provides a possible notion of geodesic extension and which is given by

$$\inf_{\mu\in\mathcal{P}_2(\mathbb{R}^d)} \left\{ \frac{W_2^2(\mu,\nu_1)}{2(t-1)} - \frac{W_2^2(\mu,\nu_0)}{2t} \right\}, \tag{$\mathcal{P}$}$$

where $t > 1$. This was used by the authors in [1] in order to construct a second order consistent discretization of Wasserstein gradient flows. We will present several different equivalent formulations of problem $(\mathcal{P})$, and a convergent numerical approach to compute its solutions.

[1] T. Gallouët, A. Natale, G. Todeschi. *From geodesic extrapolation to a variational BDF2 scheme for Wasserstein gradient flows.* Mathematics of Computation, 2024.

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